How High is Your Ceiling?

Teacher Expectations & Pedagogy from a Singapore Garden

Chris Shore, Curtis Center UCLA, Mar 2022
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Chris' Takeaway from Singapore Schools

Q: What puts Singapore so far ahead?
A: Expectations!

- Problem Solving on the Test
- Extremely High Rigor
- Clear Target (prep book)
- For All Students

The Sweet 16 Pedagogy Hits

1. Dual Objectives
2. Warm-Up on Prerequisite Skills & Numeracy
3. Tasks! (60% of the time)
4. Group Work & Norms
5. No-Options Engagement
6. Concepts-Procedures-Applications (C-P-A Progression)
7. Manipulatives & Measurement
8. Structured Notes with Feedback
9. Lapboards
10. Chunking
11. Higher-Order Thinking Questions
12. Use Student Thinking
13. Differentiation by Extension
14. Gradual Reel-In
15. Instructional Technology
16. Reflective Conclusion

CCSS.7.G.B.4

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Related Reading:

- *Principles to Action,* NCTM
- *Building a Thinking Classroom,* Dr. Peter Liljedahl
- *SMP Posters by MPJ*, The Math Projects Journal
- *It Takes Two Hands to Clap* (forthcoming blog post of Chris' Visit to Singapore Schools)

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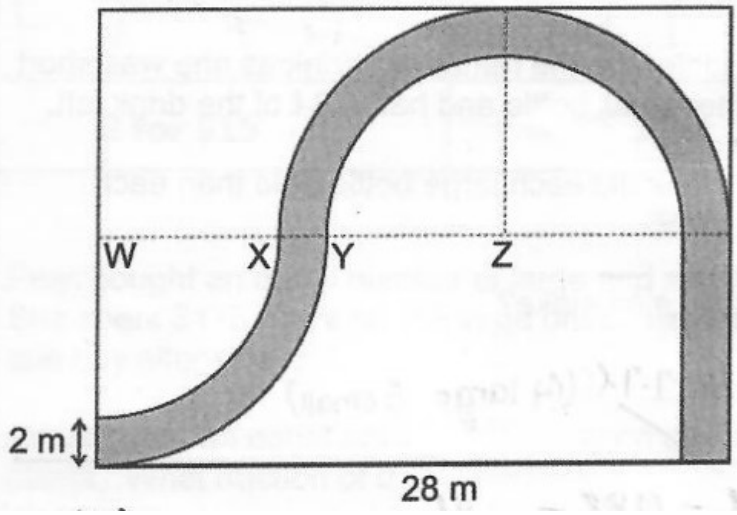
The Singapore Garden Problem

Sample of PSLE (Primary School Leave Exam), equivalent to E.O.C. in U.S. 7th Grade.

14 The figure shows a path of width 2 m in a rectangular garden of length 28 m. The outline of the path is made up of quarter circles with centre W, semicircles with centre Z and straight lines. WX = YZ.

(a) What is the width of the rectangular garden?

(b) Find the area of the path. Take $\pi = 3.14$.



1) Solve the sample exam prep problem.

Sample PSLE Problem (cont'd)

2) Compare and contrast the content being tested in Singapore to that of the U.S.

3) What problem solving skills are being tested in this example?

4) How often are our students presented with this kind of assessment rigor?

5) How would this level of expected rigor change your teaching?

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Our Circle Lesson, Part 1: Slice of Pi

You will recognize a pattern in the attributes of circles to determine the definition of pi.

Warm-up:

a) Draw a rectangle with width of 1.5 inches and a length of $1\frac{3}{8}$.

b) Write the decimal equivalent for: $\frac{1}{2}, \frac{1}{4}, \mathbf{\frac{1}{8}}, \frac{3}{4}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$

c) What is pi?

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1. Measure the circumference and the diameter of your circular object. Record those values and the ratio of the circumference to the diameter on the Class Google Sheet, as well as in the chart below.

Name	units	C	d	C/d
You				
A Classmate				
Another Classmate				

2. Do you notice anything interesting in the chart?

3. What is π?

Your Wrinkle Sprinkle:

Our Circle Lesson, Part 2: Slices of Pi (with rectangle a la mode)

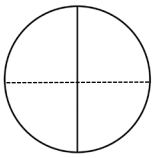
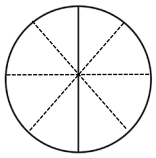
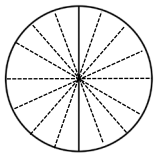
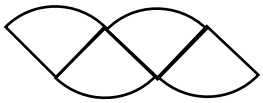
You will think abstractly to derive the formula for the area of a circle.

Warm-up:

a) What is the area of a 3' x 6' rectangle?

b) What is the area of a parallelogram with a height of 3 units and a base of 6 units?

4. Cut out the circle provided to you on the next page.
   a) What is the radius of your circle? _____
   b) What is the *measured* circumference of your circle? _____
   c) What is the *calculated* circumference of your circle? _____
5. Let's derive the formula for the area of a circle by cutting it up into pieces.
   a) Start by cutting our circle into quarters and place it on your desk as shown.

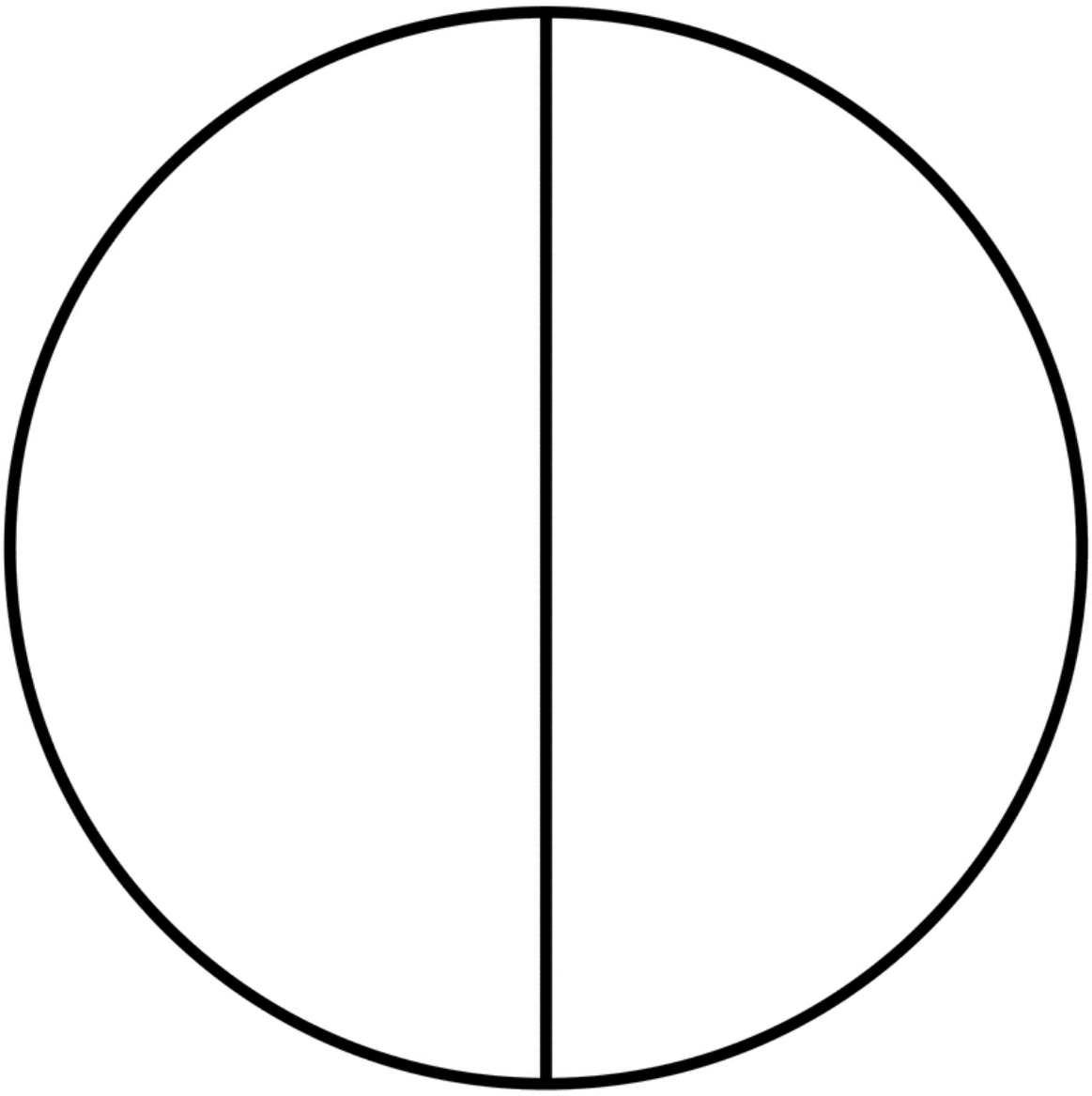





A ≈ _____ A ≈ _____ A ≈ _____

   b) Measure the length and height of the new figure; use these values to approximate the area of the imaginary rectangle encompassing the figure.

6. Watch the following video to see how this process would play out if we continued to cut thinner and thinner slices. bit.ly/AreaCircleDerived
   a) If you could cut infinitely many slices that are infinitely thin, as in the video demonstration, what would be the area of your eventual rectangle? _____
   b) What is the formula for the area of any circle? _____
   c) Use this formula to calculate the area of your circle. _____
   How does this area relate to the area of your theoretical rectangle?

Your Wrinkle Sprinkle:



Our Circle Lesson, Part 3: Notes on the Area of Circles

You will calculate the exact and approximate values for the area of a circle.

Warm-up: Draw a circle; label its diameter & radius.

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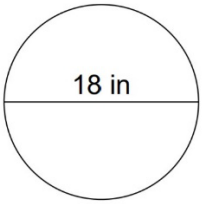
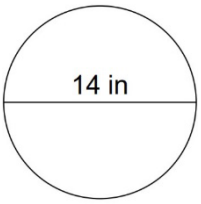
7. a) For the example on the left, watch your teacher find the area the circle.
   b) After they are finished, write your notes on the example.
   c) After your notes on the first example are completed, attempt the example on the right on your whiteboard (next page).
   d) Once your response is confirmed as correct, copy it to your notes.





8. Teachers: With what other types of examples do you want your students to possess fluency?

Your Wrinkle Sprinkle:

Simulated Whiteboard

Our Circle Lesson, Part 4: The Circular Sandbox Problem

You will solve problems involving the area of a circle.

Warm-up: What is the area of a circle with a diameter of 5 inches?

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9. Jackson is building a circular sandbox for his children that has a wooden border around it as shown in the picture below. He wants the top, horizontal portion, which he calls the bench, painted red. The inner diameter of the sandbox is 10 feet, and the width of the bench is 2 feet.

   a. What is the area of wood (in square feet) which needs to be painted red?

   b. If one quart of paint covers 100 square feet and cost $4, how much will Jackson pay for the red paint?

Your Wrinkle Sprinkle:

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The Top Hits for Math Pedagogy

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10. Which of NCTM's 8 Teaching Practices from *Principles to Action* do you see on this list?

11. Which of Dr. Liljedahl 's 14 Elements from *Building a Thinking Classroom* do you see on this list?

12. Which of Dr. Perry's 7 *Features of an Equitable Classroom* do you see on this list?

13. Which ONE of these Pedagogical Hits are you going to implement in your classroom in the next 2 weeks?

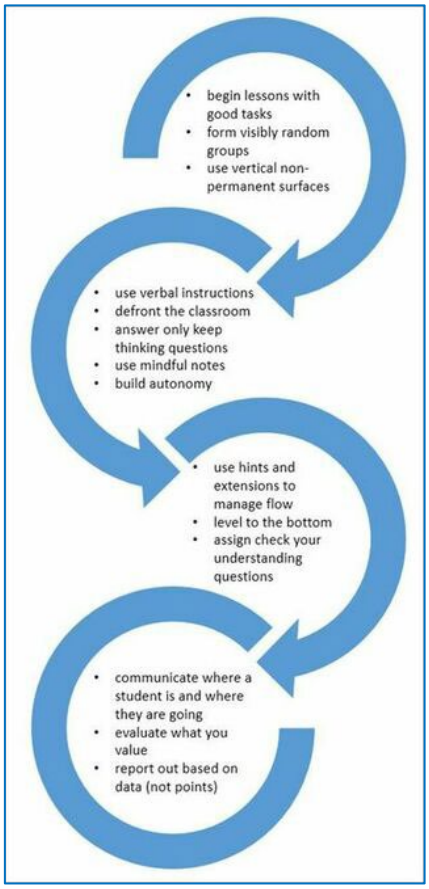
Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Principles to Action, NCTM

7 Features of An Equitable Classroom

1. High-Cognitive Demand Tasks
2. Clear Expectations
3. Provide Resources
4. Students Share Work
5. Student Justify & Explain
6. Answer Questions with Questions
7. Model Reasoning

Dr. Ayanna Perry, NCTM



Building a Thinking Classroom, Dr. Peter Liljedahl

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Group Norms

1. I will share my thinking. (All Voices)
2. I will listen to the thinking of others. (All Present)
3. I will speak up if I disagree or don't understand. (All In)