

Polynomial Farm & the C-P-A Progression

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The Math Projects Journal
Murrieta Valley USD, CA



@MathProjects



Embrace our differences to the 3rd degree!



Polynomial Farm & the C-P-A Progression

$$x + x = ?$$

A) $2x$

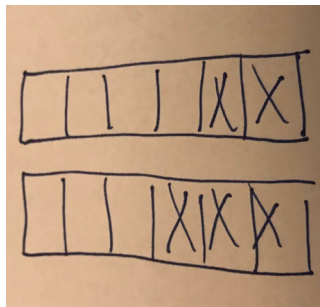
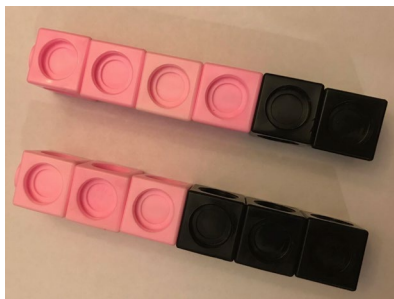
B) x^2

How do you know?



The C-P-A Progression

Concrete → Representational → Abstract



$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$

Concrete → Pictorial → Abstract



Conceptual → Procedural → Application

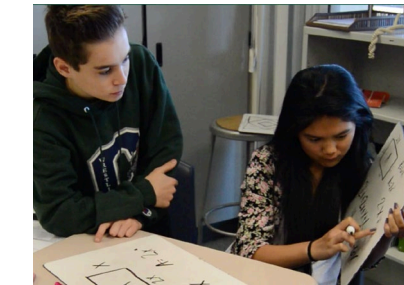
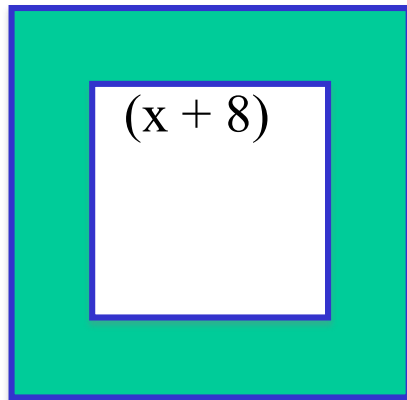


The C-P-A Progression

Goals for today

Conceptual → Procedural → Application

$$(2x - 6)$$

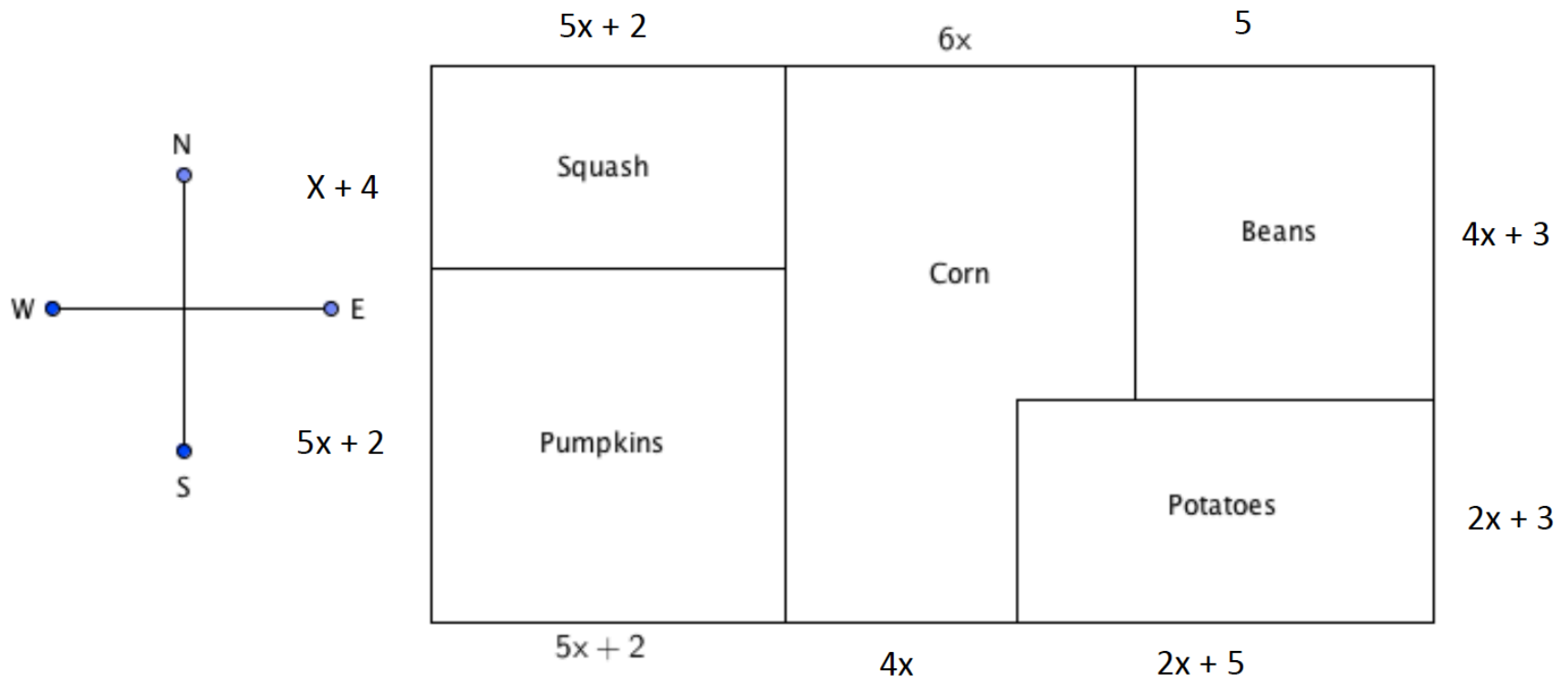


Amount	Ingredient
1 cup	flour
1 cup	yellow cornmeal
4 teaspoons	baking powder
$\frac{3}{4}$ teaspoon	salt
$\frac{3}{4}$ cup	sugar
2	eggs
1 cup	milk



Polynomial Farm

Conceptual → Procedural → Application



Polynomial Farm Day 1... & 2

Conceptual → Procedural → Application

	4in.	5in.
2in.	A	B
3in.	C	D

	2x	3
x	A	B
1	C	D

	Perimeter		Area	
A	12	6x	8	2x ²
B	14	2x + 6	10	3x
C	14	4x + 2	12	2x
D	16	8	15	3
Large Rectangle	20 28	6x + 8	45	2x ² + 5x + 3

$$(x + 1)(2x + 3)$$



Polynomial Farm

Video Example of Group Work & Student Discourse



**Group
Expectations**



**Group
Dynamics**

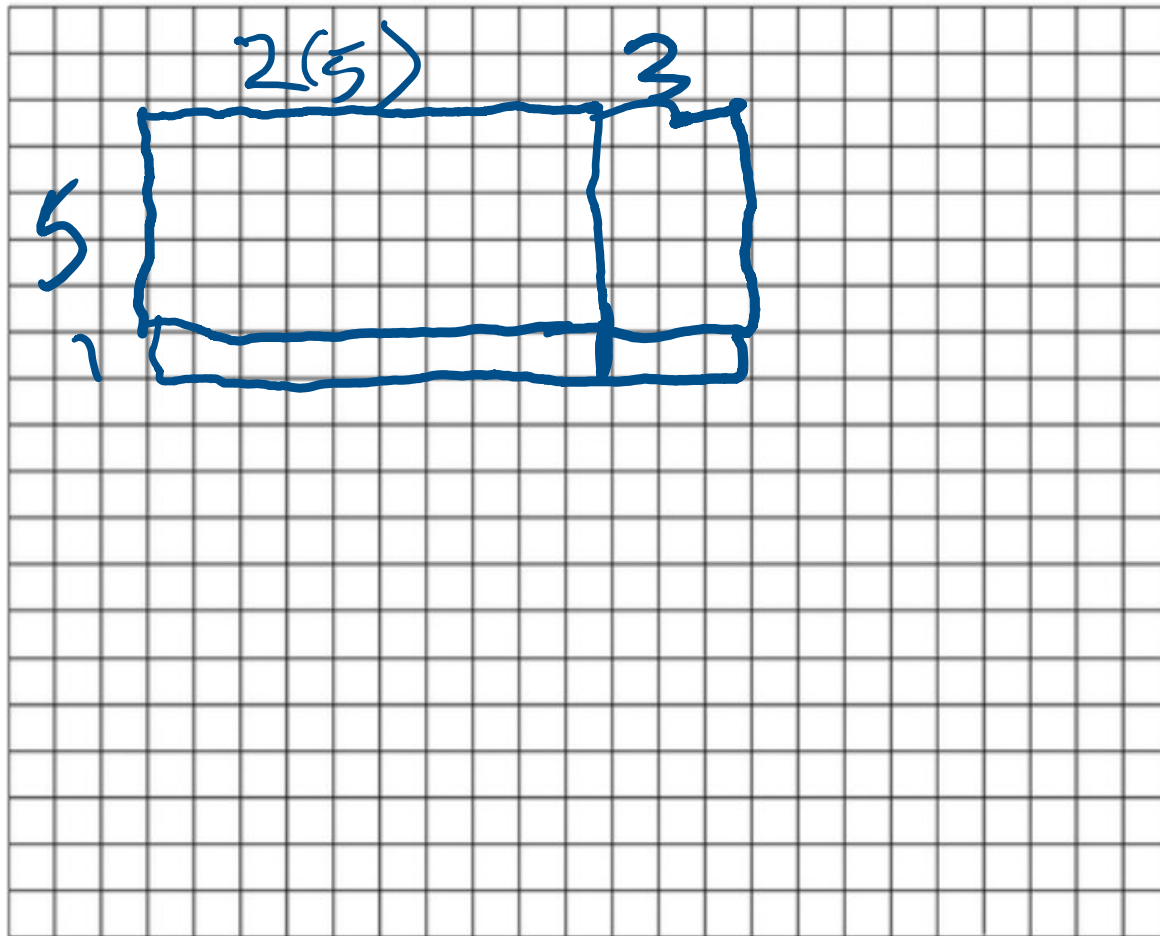


**Passionate
Argument**



Polynomial Farm Day 3

Conceptual \rightarrow Procedural \rightarrow Application

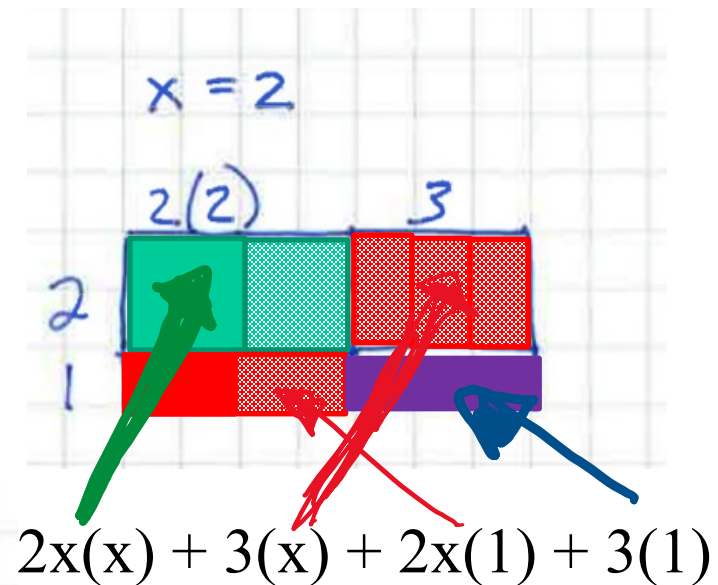
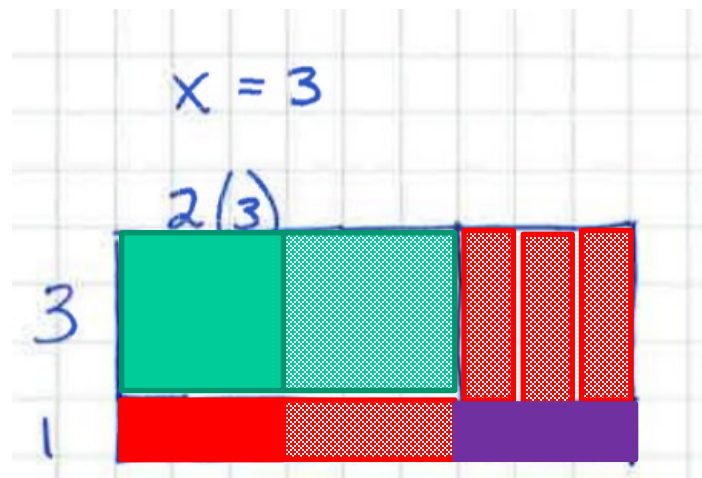
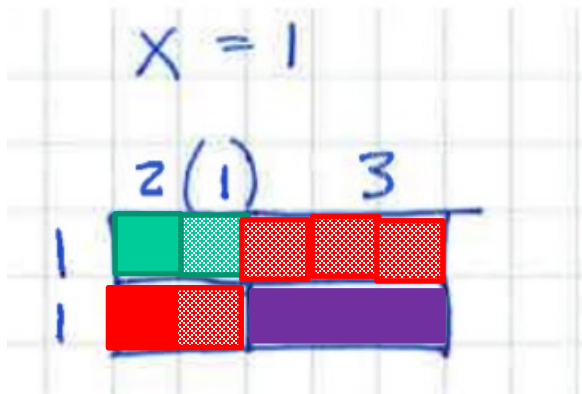


$$x = 5$$



Polynomial Farm Day 3

Conceptual \rightarrow Procedural \rightarrow Application

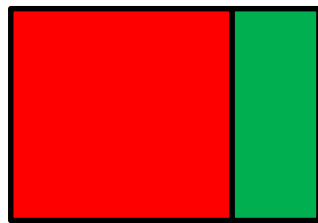
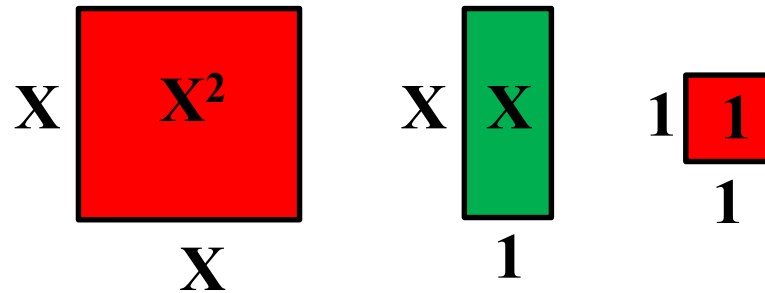


$$2x^2 + 5x + 3$$

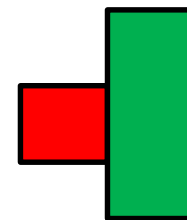
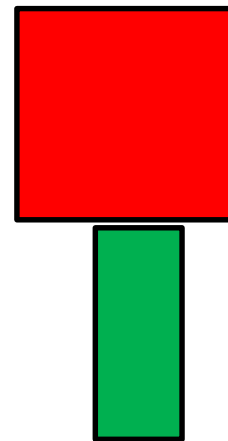


Polynomial Farm Day 4

Introduction of Algebra Tiles



Okay



Not
Okay



x^2

Polynomial Farm Day 4

 x

1

Conceptual \rightarrow Procedural \rightarrow Application



$$(x + 1)(2x + 3) = 2x^2 + 5x + 3$$



Polynomial Farm Day 5

Conceptual → Procedural → Application

Group Quiz

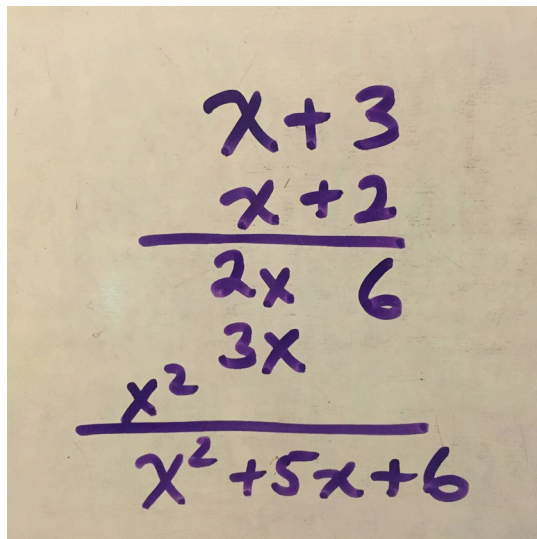
**Adding, Subtracting & Multiplying
Polynomials**



Polynomial Farm, Day 6

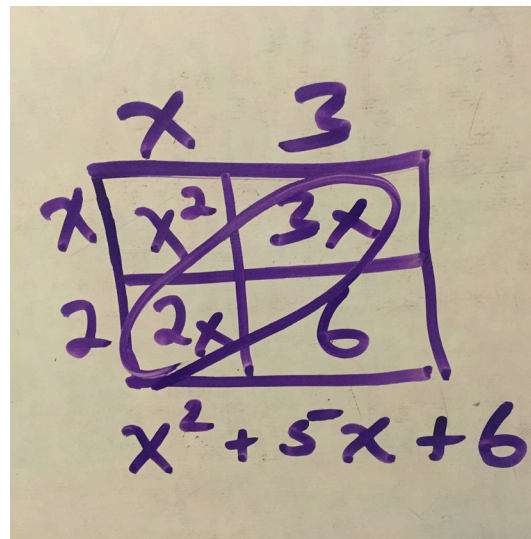
Conceptual → Procedural → Application

Stacking Method



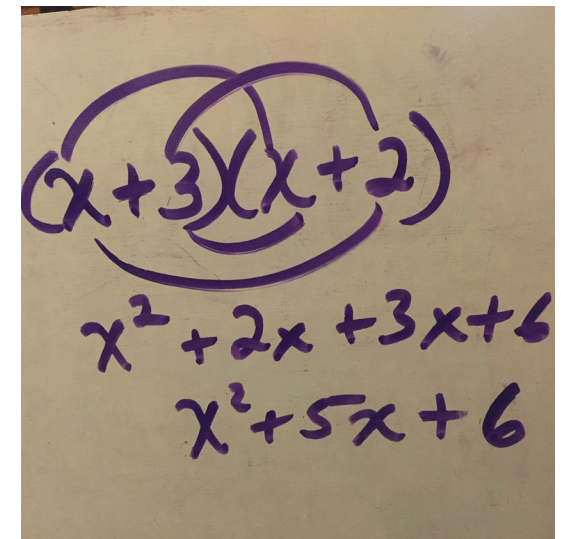
Handwritten work showing the stacking method for multiplying $(x+3)$ and $(x+2)$. The first polynomial $x+3$ is written above the second $x+2$. A horizontal line separates them. Below the line, the products $2x$ and 6 are written. Another horizontal line is drawn. Below this line, the products x^2 and $3x$ are written. A final horizontal line is drawn, and the sum $x^2 + 5x + 6$ is written below it.

Area Model
Box Method




Handwritten area model for multiplying $(x+3)$ and $(x+2)$. A rectangle is divided into four smaller rectangles. The top-left rectangle is labeled x^2 , the top-right is $3x$, the bottom-left is $2x$, and the bottom-right is 6 . The sides of the large rectangle are labeled x and 3 on top, and x and 2 on the left. A diagonal line is drawn from the bottom-left to the top-right. Below the rectangle, the sum $x^2 + 5x + 6$ is written.

FOIL method

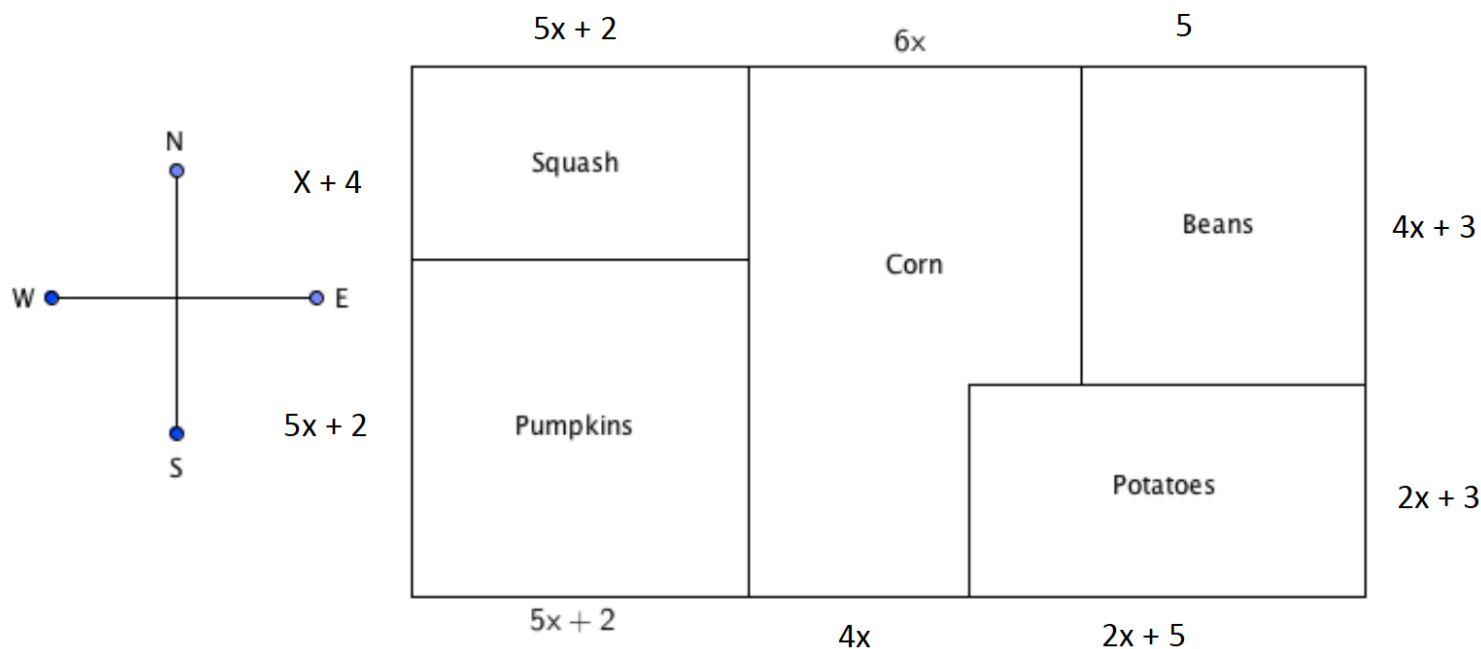


Handwritten work showing the FOIL method for multiplying $(x+3)$ and $(x+2)$. The two polynomials are circled and written as $(x+3)(x+2)$. Below this, the terms are expanded: $x^2 + 2x + 3x + 6$. A final line shows the simplified sum $x^2 + 5x + 6$.


$$(x + 2)(x + 3) = x^2 + 5x + 6$$

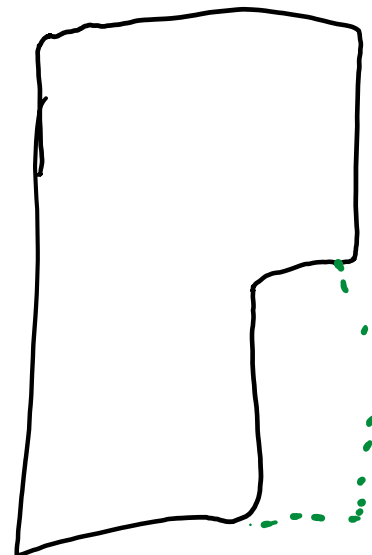
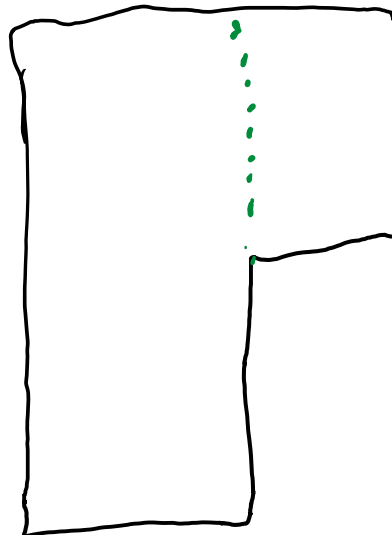
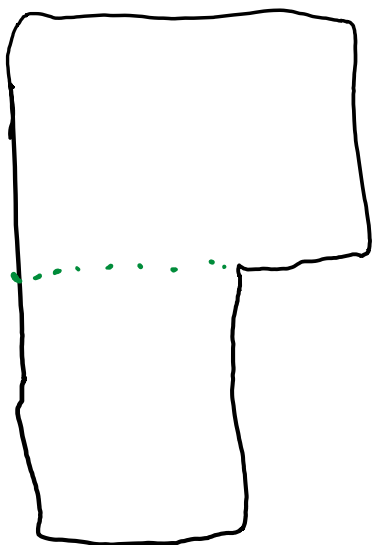
Polynomial Farm, Day 7

Conceptual → Procedural → Application



Polynomial Farm, Day 7

Conceptual → Procedural → Application



Polynomial Farm, Day 8 & 9

Conceptual → Procedural → Application

Day 8:
Support Strategies

$$(3x + 5)^2$$

$$(a + b)(a - b)$$

$$(x + 5)(x^2 + 2)$$

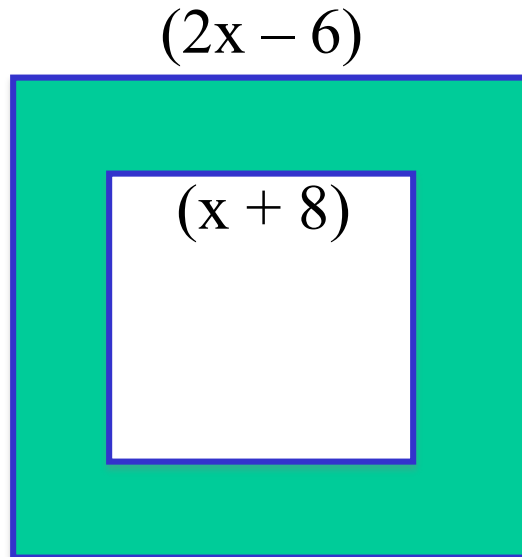
$$2x(x + 4) + (x + 5)(3x - 2)$$

Day 9
Notes & Practice



Polynomial Unit, Decision Time

Audra is framing a square painting with side lengths of $(x + 8)$ inches. The total area of the painting and the frame has a side length of $(2x - 6)$. The material for the frame is \$0.10 per square inch.

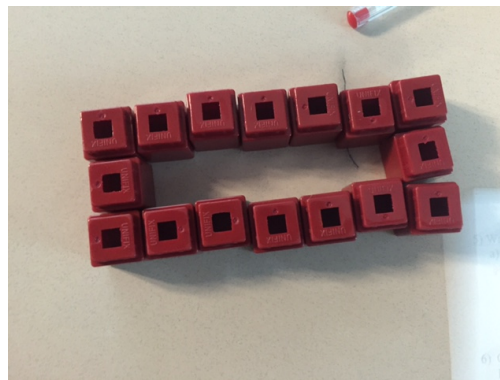
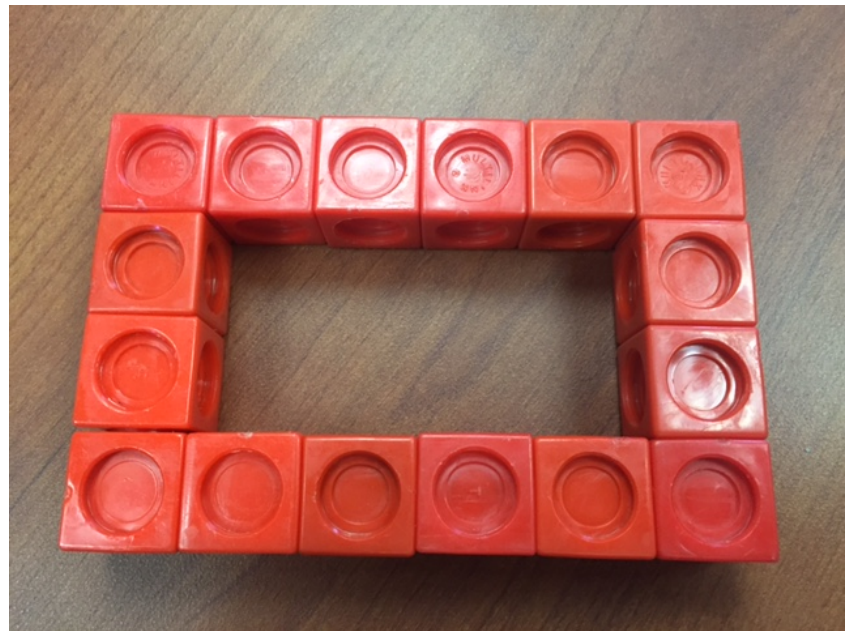
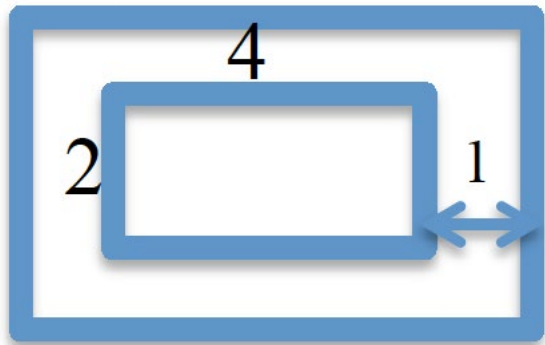


$$(2x - 6)^2 - (x + 8)^2$$

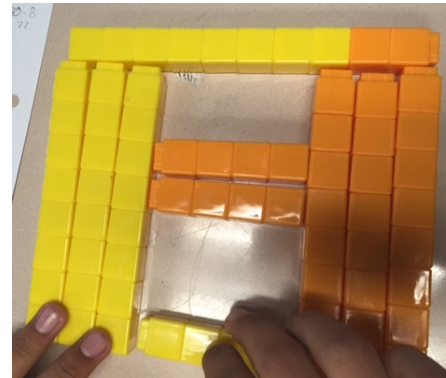
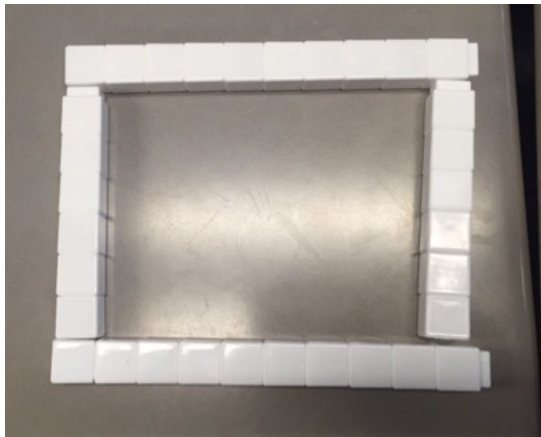
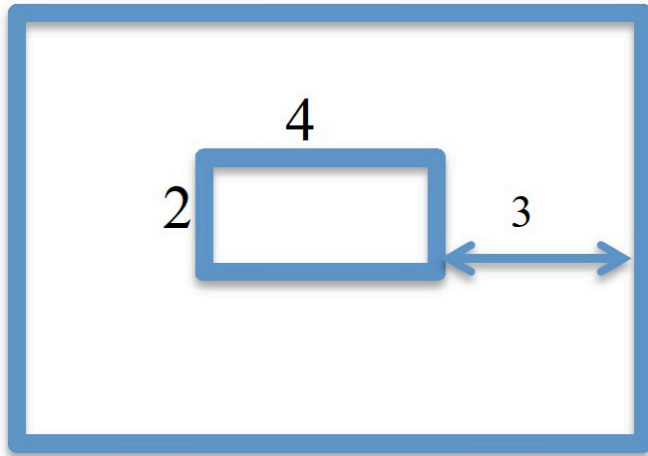
1. Write the expression for the area of the painting.
2. Write the expression for the area of the painting and the frame.
3. Write the expression for the area of the frame.
4. Find the area of the frame if $x=16$.
5. Find the cost of the material for the frame.



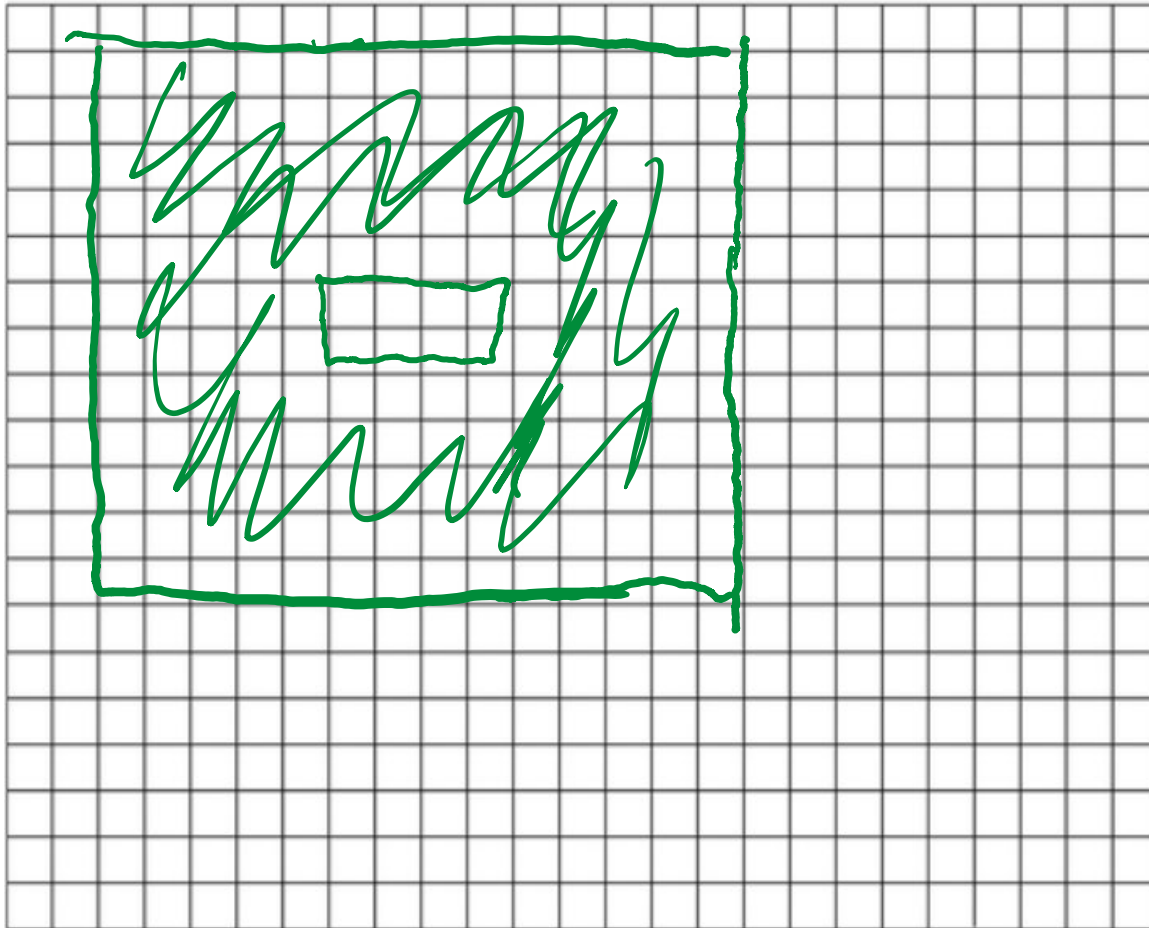
Polynomial Unit, Day 10



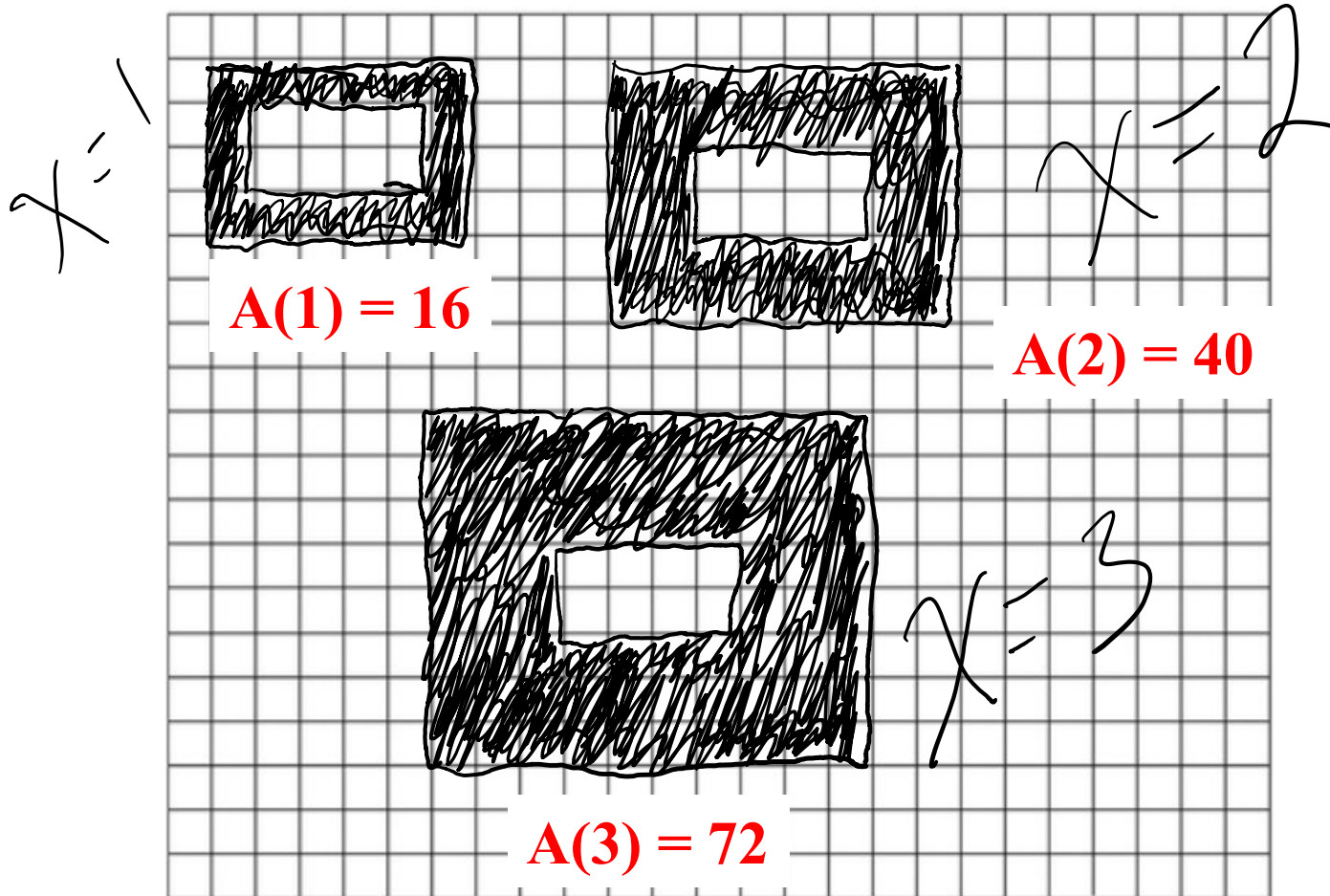
Polynomial Unit, Day 10



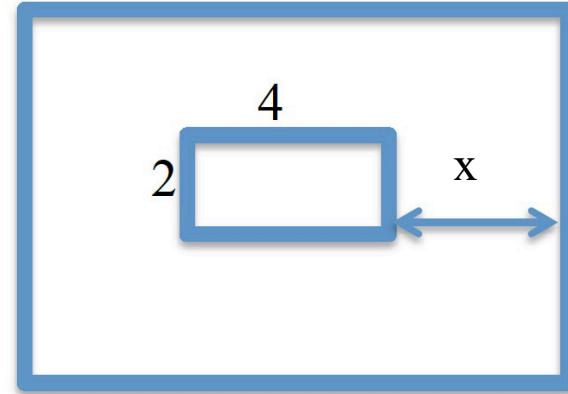
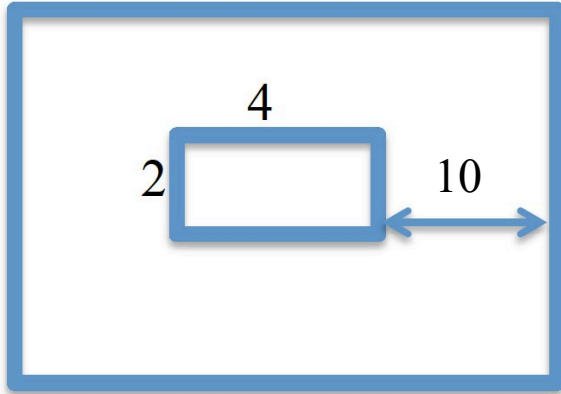
Polynomial Unit, Day 11



Polynomial Unit, Day 11



Polynomial Unit, Day 11



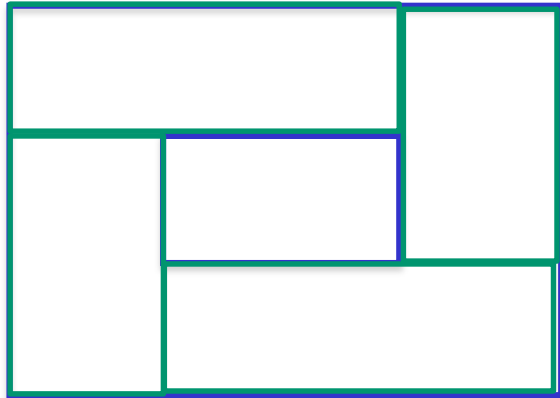
x^2	$4x$	x^2
$2x$		$2x$
x^2	$4x$	x^2

$$4x^2 + 2(2x) + 2(4x)$$



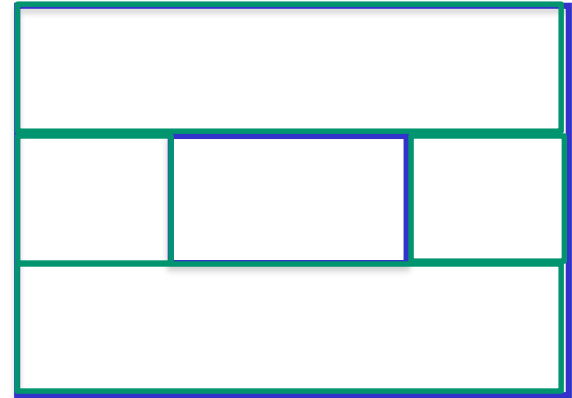
Polynomial Unit, Day 11

$$4x^2 + 2(2x) + 2(4x)$$

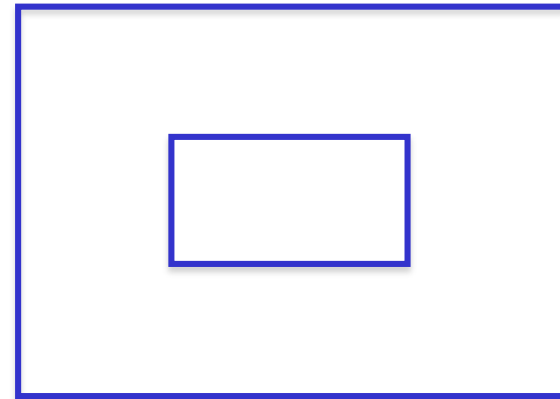


$$2x(x+4) + 2x(x+2)$$

$$4x^2 + 12x$$



$$2x(2x+4) + 2(2x)$$



$$(2x+4)(2x+2) - (2)(4)$$



Polynomial Unit, Day 11

$$A(x) = 4x^2 + 12x$$

$$A(1) = 4(1)^2 + 12(1) = 16$$

$$A(2) = 4(2)^2 + 12(2) = 40$$

$$A(3) = 4(3)^2 + 12(3) = 72$$

$$A(10) = ?$$



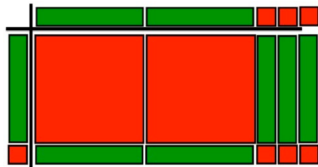
Polynomial Unit, Day 12-15

- Notes & Practices
- Group Quiz
- Review
- Test

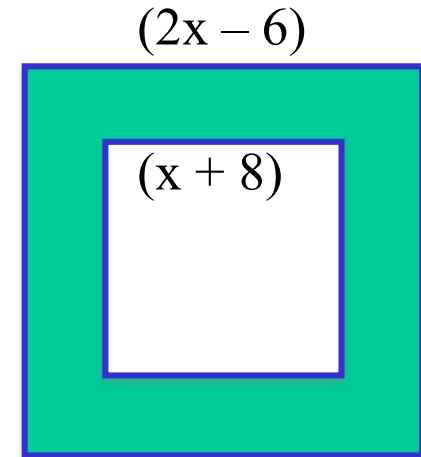
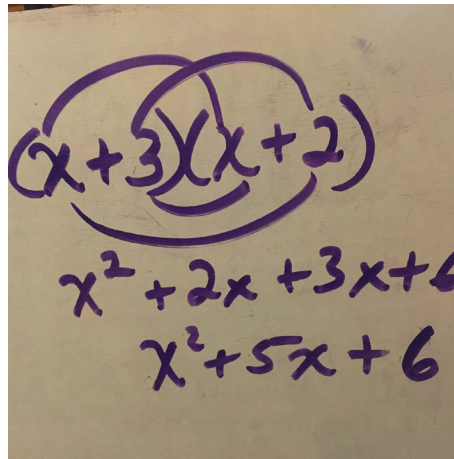
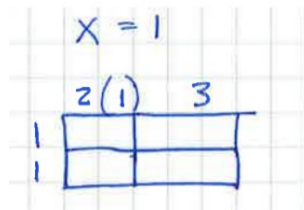
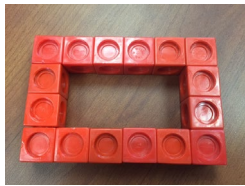


Polynomial Unit Results

Conceptual → Procedural → Application
Practice



	2x	3
x	A	B
1	C	D



Lowest Grade on District Benchmark for Polynomials = **76%**



C-P-A Progression in 1st Grade

Subtract multiples of 10 in the range 10-90 using concrete models or drawings.

Conceptual Task

Practice Standard: Recognizing Patterns

Taking 10 Away

1. Build 98 with base-10 blocks.
2. Subtract 10 and say the number sentence out loud.
3. Keep subtracting 10 until you get as close to 0 as possible.
4. What pattern did you notice while subtracting?



C-P-A Progression in 1st Grade

Subtract multiples of 10 in the range 10-90 using concrete models or drawings.

Application Tasks

Practice Standard: Solving Problems



Beads and Stickers

- 1) Marta is stringing beads on a necklace. She has 34 beads. She used 20 for her necklace. How many beads does she have left?
- 2) Bill had 87 stickers. He used 50 stickers in his class. How many stickers did he have left?



C-P-A Progression in 4th Grade

Multiply a fraction by a whole number. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.

Conceptual Tasks

Practice Standard: Seeing Structure

Party Snacks

- Coach Ashley is giving Gatorade to her team of 18 players. If each player drank $\frac{1}{4}$, how much Gatorade did the team drink?



C-P-A Progression in 4th Grade

Multiply a fraction by a whole number. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.

Application Tasks

Practice Standard: Seeing Structure

Making Muffins

Your corn muffin recipe makes 9 muffins, but you want to make 18 muffins. How much of each ingredient is needed to make 18 muffins?

Amount	Ingredient
1 cup	flour
1 cup	yellow cornmeal
4 teaspoons	baking powder
$\frac{3}{4}$ teaspoon	salt
$\frac{3}{4}$ cup	sugar
2	eggs
1 cup	milk



Why the **C-P-A** progression?

From NCTM

FOCUS

“...teachers must focus on the mathematical ideas embedded within the standards. Attention should not be limited to the development of **procedural** skill. Rather, **conceptual** understanding along with **application** of mathematical ideas should play a key role in students’ learning.”

Students need it to be good mathematicians.



Why the **C-P-A** progression?

Assessments



Notes-Oriented

40%



Task-Oriented

60%

Students need it for their state assessments.



Why the **C-P-A** progression?

Students will need it in the work force.

Critical Thinking, Communication, Collaboration, Creativity

Top 10 skills



in 2020

1. Complex Problem Solving
2. Critical Thinking
3. Creativity
4. People Management
5. Coordinating with Others
6. Emotional Intelligence
7. Judgment and Decision Making
8. Service Orientation
9. Negotiation
10. Cognitive Flexibility

in 2015

1. Complex Problem Solving
2. Coordinating with Others
3. People Management
4. Critical Thinking
5. Negotiation
6. Quality Control
7. Service Orientation
8. Judgment and Decision Making
9. Active Listening
10. Creativity



Clothesline Math

Call to Action

Fail Grandly



No Real Risk



2-Week Rule

10%

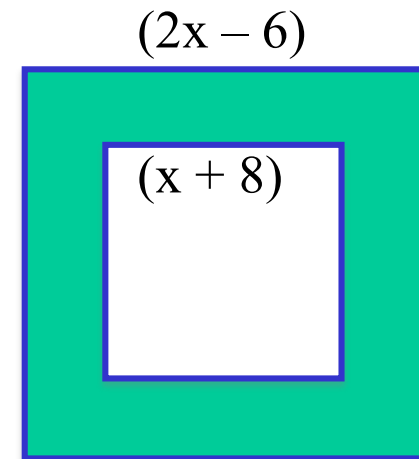
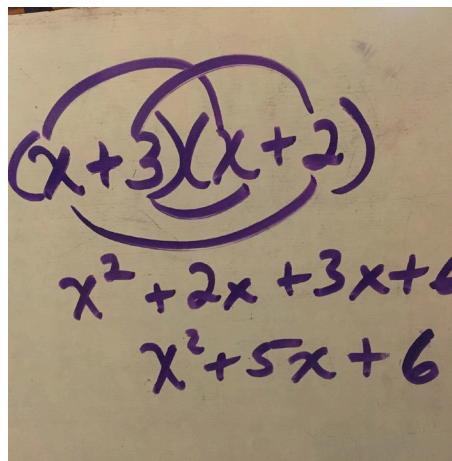
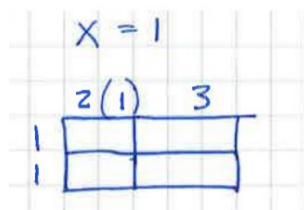
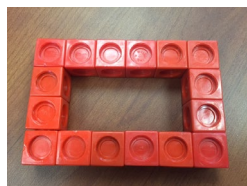


Teach through the C-P-A Progression...

...with the faith that they can learn it,
and that we can teach it to them,



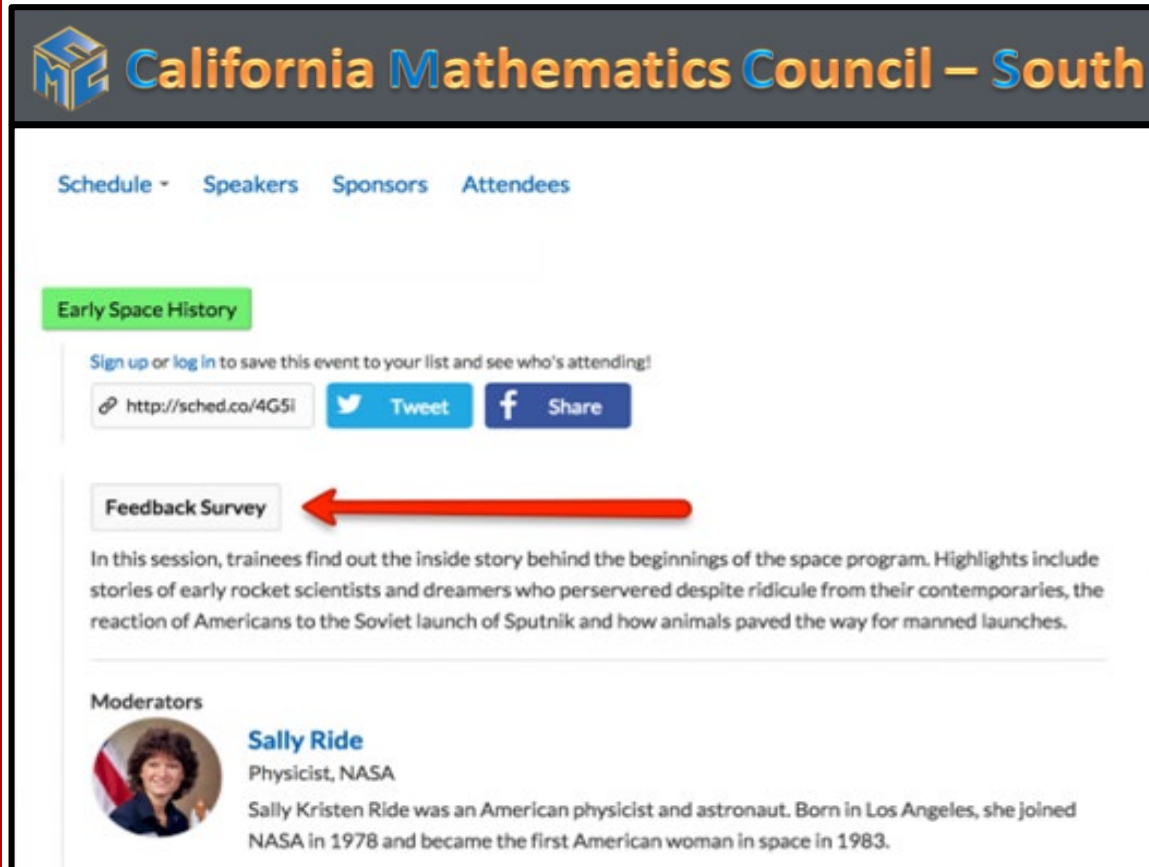
	2x	3
x	A	B
1	C	D



...so you can change this world,
one math lesson at a time.



Speaker Evaluation






California Mathematics Council – South

Schedule • Speakers Sponsors Attendees

Early Space History


Sign up or log in to save this event to your list and see who's attending!

<http://sched.co/4G5I>  Tweet  Share

Feedback Survey 

In this session, trainees find out the inside story behind the beginnings of the space program. Highlights include stories of early rocket scientists and dreamers who persevered despite ridicule from their contemporaries, the reaction of Americans to the Soviet launch of Sputnik and how animals paved the way for manned launches.

Moderators

 **Sally Ride**
Physicist, NASA

Sally Kristen Ride was an American physicist and astronaut. Born in Los Angeles, she joined NASA in 1978 and became the first American woman in space in 1983.

In the app
*Go to this session's
page, then click
the “Feedback
Survey” button*

