

Polynomial Farm & the C-P-A Progression

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The Math Projects Journal
Murrieta Valley USD, CA

@MathProjects

P 14 & a pencil

Embrace our differences to the 3rd degree!

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Polynomial Farm & the C-P-A Progression

$x + x = ?$

A) $2x$ B) x^2

How do you know?

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The C-P-A Progression

Concrete → Representational → Abstract

Concrete → Pictorial → Abstract

Conceptual → Procedural → Application

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Polynomial Farm

Conceptual → Procedural → Application

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Polynomial Farm Day 1... & 2

Conceptual → Procedural → Application

4in.	5in.		2x	3
2in.	A	B	A	B
3in.	C	D	C	D

	Perimeter	Area
A	12	6x
B	14	2x + 6
C	14	4x + 2
D	16	8
Large Rectangle	28	6x + 8

(x + 1)(2x + 3)

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Polynomial Farm

Video Example of Group Work & Student Discourse

Group Expectations

Group Dynamics

Passionate Argument

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Polynomial Farm Day 3

Conceptual → Procedural → Application

$x=5$

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Polynomial Farm Day 3

Conceptual → Procedural → Application

$2x^2 + 5x + 3$

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Polynomial Farm Day 4

Introduction of Algebra Tiles

Okay Not Okay

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Polynomial Farm Day 4

Conceptual → Procedural → Application

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Polynomial Farm Day 5

Conceptual → Procedural → Application

Group Quiz
Adding, Subtracting & Multiplying
Polynomials

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Polynomial Farm, Day 6

Conceptual → Procedural → Application

Stacking Method	Area Model Box Method	FOIL method

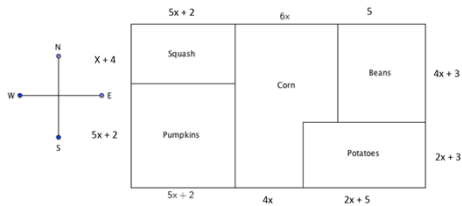
$(x + 1)(2x + 3) = 2x^2 + 5x + 3$

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Polynomial Farm, Day 7

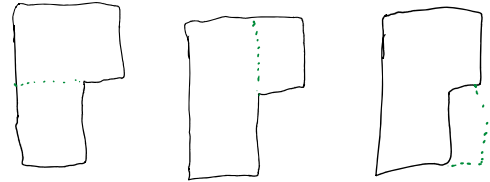
Conceptual → Procedural → Application



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Polynomial Farm, Day 7

Conceptual → Procedural → Application



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Polynomial Farm, Day 8 & 9

Conceptual → Procedural → Application

$$(3x + 5)^2$$

$$(a + b)(a - b)$$

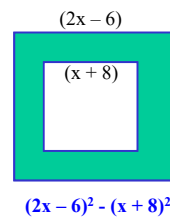
$$(x + 5)(x^2 + 2)$$

$$2x(x + 4) + (x + 5)(3x - 2)$$

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Polynomial Unit, Decision Time

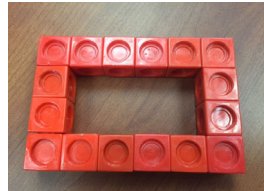
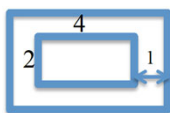
Audra is framing a square painting with side lengths of $(x + 8)$ inches. The total area of the painting and the frame has a side length of $(2x - 6)$. The material for the frame is \$0.10 per square inch.



1. Write the expression for the area of the painting.
2. Write the expression for the area of the painting and the frame.
3. Write the expression for the area of the frame.
4. Find the area of the frame if $x = 16$.
5. Find the cost of the material for the frame.

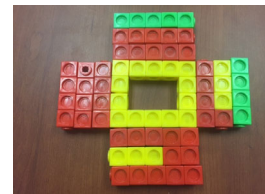
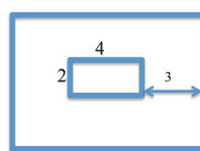
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Polynomial Unit, Day 10



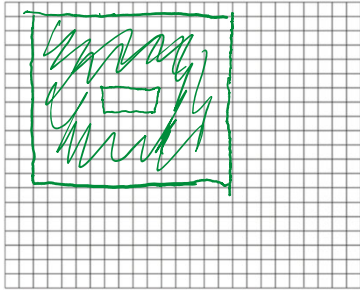
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Polynomial Unit, Day 10



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Polynomial Unit, Day 11

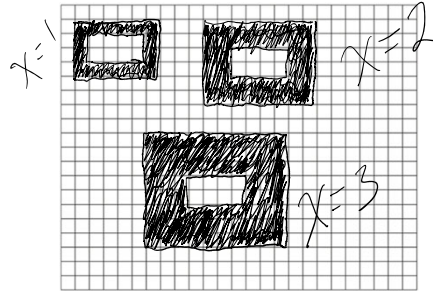


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Polynomial Unit, Day 11



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Polynomial Unit, Day 12-15

- Notes & Practices
- Group Quiz
- Review
- Test

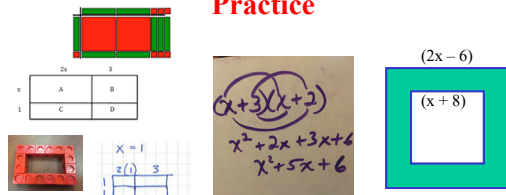
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Polynomial Unit Results

Conceptual → Procedural → Application
Practice



Lowest Grade on District Benchmark for Polynomials = **76%**

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C-P-A Progression in 1st Grade

Subtract multiples of 10 in the range 10-90 using concrete models or drawings.

Conceptual Task

Practice Standard: Recognizing Patterns

Taking 10 Away

1. Build 98 with base-10 blocks.
2. Subtract 10 and say the number sentence out loud.
3. Keep subtracting 10 until you get as close to 0 as possible.
4. What pattern did you notice while subtracting?

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C-P-A Progression in 1st Grade

Subtract multiples of 10 in the range 10-90 using concrete models or drawings.

Application Tasks

Practice Standard: Solving Problems

Beads and Stickers

- 1) Marta is stringing beads on a necklace. She has 34 beads. She used 20 for her necklace. How many beads does she have left?
- 2) Bill had 87 stickers. He used 50 stickers in his class. How many stickers did he have left?

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C-P-A Progression in 4th Grade

Multiply a fraction by a whole number. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

Conceptual Tasks

Practice Standard: Seeing Structure

Party Snacks

- Coach Ashley is giving Gatorade to her team of 18 players. If each player drank $1/5$, how much Gatorade did the team drink?

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C-P-A Progression in 4th Grade

Multiply a fraction by a whole number. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

Application Tasks

Practice Standard: Seeing Structure

Making Muffins

Your corn muffin recipe makes 9 muffins, but you want to make 18 muffins. How much of each ingredient is needed to make 18 muffins?

Amount	Ingredient
1 cup	flour
1 cup	yellow cornmeal
4 teaspoons	baking powder
$\frac{3}{4}$ teaspoon	salt
$\frac{1}{4}$ cup	sugar
2	eggs
1 cup	milk
$\frac{3}{4}$ cup	vegetable oil

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Why the C-P-A progression?

From NCTM

FOCUS

"...teachers must focus on the mathematical ideas embedded within the standards. Attention should not be limited to the development of **procedural** skill. Rather, **conceptual** understanding along with **application** of mathematical ideas should play a key role in students' learning."

Students need it to be good mathematicians.

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Why the C-P-A progression?

Assessments



Notes-Oriented
40%



Task-Oriented
60%

Students need it for their state assessments.

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Why the C-P-A progression?

The 4C's:

Critical Thinking, Communication, Collaboration, Creativity
Top 10 skills

in 2020

1. Complex Problem Solving
2. Critical Thinking
3. Creativity
4. People Management
5. Coordinating with Others
6. Emotional Intelligence
7. Judgment and Decision Making
8. Service Orientation
9. Negotiation
10. Cognitive Flexibility

in 2015

1. Complex Problem Solving
2. Coordinating with Others
3. People Management
4. Critical Thinking
5. Negotiation
6. Quality Control
7. Service Orientation
8. Judgment and Decision Making
9. Active Listening
10. Creativity



Students will need it in the work force.

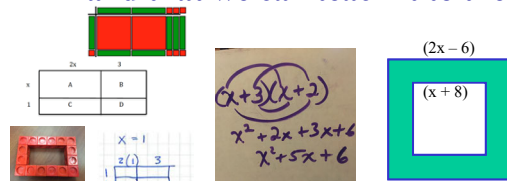
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Teach through the C-P-A Progression...

...with the faith that they can learn it, and that we can teach it to them,



...so you can change this world, one math lesson at a time.

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