PROJECT

4-DIGIT PROBLEM
BY COLLEGE PREPARATORY MATHEMATICS; SACRAMENTO, CALIFORNIA

THE ASSIGNMENT
Arrange four 8's to make each integer from 1 through 5. You are permitted to use any of the four operations (addition, subtraction, multiplication and division) as well as parentheses, roots and exponents. You may also use any number you wish for an exponent or root, but other than that, only 8's are allowed. For instance, 8^2 is permissible, but 2 • 8 is not. The number 88 may count as two 8's, but the number 18 is not permissible. You must use exactly four 8's for each solution. For example: $88/8 + 8 = 19$.

LESSON PLAN
Begin the class with the following warm-up activity. Show them how $8 + 8 + 8 + 8 = 32$ and $8 + 8 • 8 + 8 = 80$. Then ask them to arrange four 8's to produce 19, using any or all of the four operations. You may get several like the example given above. Then write the problem $8 + 88/8$ on the board and ask your students to simplify it. Many students may erroneously give the answer as 12 (the real answer is 19). Use this example to illustrate the importance of the Order of Operations. Then, challenge them to modify this example without changing the order of the numbers to produce 12: $(8 + 88)/8$. You can then use this example to illustrate the use of parentheses.

For each integer, there are several solutions. Keep the class discussions open to include them all, and point out the merits of each. Then, have the students create another number, like arranging four 8's to produce 15: $8 + 8 - 8 ÷ 8$. Students often like to get tricky and see who can come up with the most clever solution. After they practice, discuss with your students useful tools such as square roots, cube roots, and the exponent of zero.

> **Concepts**
> Order of operations, roots and exponents
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> **Time:** One hour
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> **Materials**
> Butcher block paper (optional)
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> **Preparation**
> Students need a general understanding of the order of operations. For the optional activity, on several sheets of butcher block paper place the numbers 1-100 in a column down the left margin. Usually, it is best to place 1-25 on the first sheet, 26-50 on the next, etc. Then hang these sheets on a wall in the room.

Helpful Hints:

$8/8 = 1$	$8 - 8 = 0$	$8 + 8 = 16$	$88/8 = 11$
$8^2 = 64$	$\sqrt[3]{8} = 2$	$8^0 = 1$	$\sqrt{8 + 8} = 4$

Once they have the idea of how the four digit problem works, challenge the students to find each of the integers 1-5. Some possible solutions are listed below.

1: $8 + 8/8 - 8$ or $88/88$ or $(8 + 8 + 8 + 8)^0$
2: $8/8 + 8/8$ or $8^0 + 8^0 + 8 - 8$
3: $88/8 - 8$
4: $(8 • 8)^{1/3} + 8 - 8$ or $8^0 + 8^0 + 8^0 + 8^0$
5: $\sqrt{(8 + 8)} + 8/8$

FURTHER INVESTIGATION
For homework you may have students find solutions for the integers 6-10. For future problems, you may use any 4 digits to produce any type of answer. For instance, arrange four 5's to produce 21. Arrange four 3's to produce a negative number, or zero, or even a fraction.

For an ultimate challenge, invite the students to find solutions to the integers 1-100. Place the butcher block paper on the wall, and write the students' solutions next to the corresponding number as they come up with them. Leave a marker handy, and challenge the students to find the rest. Allow the sheets to remain on the wall for several weeks and watch them go for it!