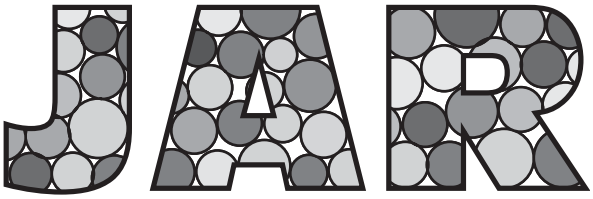
THE SHOPKEEPER'S JAR

Guess how many marbles are in the jar and win a free gift!



Submitted by Greg Rhodes, Trabuco Hills HS, Mission Viejo, CA

We have all probably seen this type of contest in a local shop. As people walk by, they usually glance at the jar, scrutinize the marbles, and take a guess. Only rarely does someone use mathematics to improve their chances of guessing the correct amount in the jar. In this lesson, students will use a variety of measuring tools to mathematically estimate the number of marbles in a given jar. The overarching principle behind this lesson is that students can estimate the number of marbles by measuring the volume of one marble (of each type if there are different sizes), measuring the volume of the jar, and dividing to get the number of marbles that can fit.

Concepts
Volume (cylinders, and spheres), circumference and diameter, measurement, estimation.

Time: 1-2 hours

Materials
Large clear jar full of marbles (the more the better) graduated cylinders, calipers (if these are unavailable, compasses will suffice), measuring tape (sewing, not hardware style).

Preparation
Establish the five different measuring stations for each of the measuring methods. Each station should have one sample of each size marble in the jar. Talk to the science teachers at your school about borrowing any equipment you may need.

LESSON PLAN

The Hook

Have the jar of marbles prominently on display, when the students enter the room. Inform them of their task and have them make an initial guess. Once the students record this guess on the handout, explain the various measurements that they are to conduct. The classroom is organized into four different measuring stations explained below.

The Measuring Stations

Students measure attributes of the marbles and the jar and make some initial calculations. The first three are dedicated to measuring the marbles (in order to find the radius and thus the volume). Station #1 has a caliper (or compass), station #2 has a measuring tape; station #3 has a graduated cylinder. For large class sizes, it is advised to have several of the measuring devices at each station. Each station should also have one of each size marble that is in the jar. Using each method of measurement, the students should measure and record the indicated dimensions and make the requested calculations. Station #4 is focused on measuring the height, circumference and diameter of the jar in order to calculate its volume.

The four stations are discussed below. The students should be allowed to move freely among the different stations to measure and re-measure at their own pace. All measurements should be made in centimeters.

#1. Volume of Marble Using Diameter **Measuring Device:** Caliper
Here the students use a caliper to measure the diameter of the marble, calculate the radius, and then calculate the volume. If you cannot find any calipers, compasses work (albeit not perfectly). If you are using compasses, have the students "pinch" the marble with the ends of the compass and then measure that distance on a ruler or measuring tape. The students should then use the diameter to calculate both circumference and volume.

#2. Volume of Marble Through Circumference **Measuring Device:** Measuring Tape
Here the students measure the circumference by wrapping the tape around the marble. Suggest to the students to measure this a few times, because it is difficult to line the tape up exactly on the "equator" of the sphere. The students record the circumference, then use it to calculate the radius, and from that, the volume.

THE SHOPKEEPER'S JAR

Guess how many marbles are in the jar and win a free gift!

LESSON PLAN (continued)

#3. Volume of Marble Through Water Displacement **Measuring Device:** Graduated Cylinder
Here the students drop the marble in a graduated cylinder containing a small amount of water. The rise in the water level is the volume of the marble measured in cubic centimeters. Also discuss how to read a graduated cylinder (measure from the base of the meniscus, the curve of the water). The students are to use the volume to calculate the radius of the marble, and from that, the circumference.

#4. Volume of Jar Through Measurement **Measuring Device:** Measuring Tape
Here the students determine the volume of the jar through whatever measurements they choose. Give very little instruction on this to give students the creative freedom to problem-solve for the necessary components to calculate volume for your specific jar. If you have any empty jars that are identical to the one filled with marbles, place them here with plenty of measuring tape.

The Empty Space

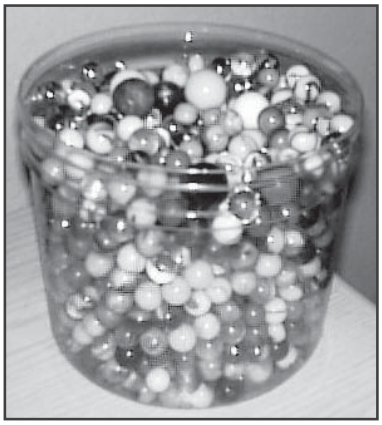
One of the richest aspects of this lesson is the dilemma of the gaps between the marbles. Have a large graduated beaker of water handy. Pour just enough water into the jar, with the marbles in it, until the marbles are immersed. Have the students record the amount of water (cubic centimeters) that is now in the jar. This is the volume of empty space between all the marbles.

The Final Count

The students now determine what they perceive to be the typical marble. Most students determine the typical marble to be the average of the three volumes or the volume of the medium size marble. By subtracting the volume of empty space from the volume of the jar, and dividing this difference by the volume of the typical marble, the students can closely estimate the number of marbles in the jar. Once they do so, it's time to drain the water out of the jar, and count the marbles. Disseminate them on paper towels among the class and have everyone make a quick count. The students should record this count and calculate their margin of error.

TEACHER COMMENTS

- This lesson works best if you have a large number of marbles in the jar (at least one hundred). Several hundred marbles can be purchased cheaply from a toy store. This lesson actually began when I found my father's old marble collection from his childhood. He had saved over one thousand marbles! Thus, the shopkeeper's jar project was born.
- Different sized marbles work just as well as identical ones. In fact, it even challenges the students to take into account different sizes. This lesson is best done with three different sizes.
- An odd-shaped jar adds even more challenge to the problem. For example, I use a licorice tub (the kind you buy at a warehouse discount store) which is not a perfect cylinder; the top is a little wider than the bottom. Interestingly, the class usually produces two strategies for determining the average diameter of the jar. The first is to average the circumference of the jar at both the top and the bottom; the second is to measure the circumference of the jar at its mid-height. Both methods are correct.
- Encourage a "science lab" atmosphere of exploration and experimenting.



This is an example of the jar or marbles that we have used in our classrooms.

THE SHOPKEEPER'S JAR

Guess how many marbles are in the jar and win a free gift!

How many marbles do you GUESS are in the jar? _____

MARBLES: Estimate the volume of each marble by using a ...

Caliper	Small	Medium	Large
Diameter/Radius	_____ /_____ **cm**	_____ /_____ **cm**	_____ /_____ **cm**
Circumference	_____ cm	_____ cm	_____ cm
Volume	_____ cc	_____ cc	_____ cc

Tape Measure	Small	Medium	Large
Diameter/Radius	_____ /_____ cm	_____ /_____ cm	_____ /_____ cm
Circumference	_____ **cm**	_____ **cm**	_____ **cm**
Volume	_____ cc	_____ cc	_____ cc

Graduated Cylinder	Small	Medium	Large
Diameter/Radius	_____ /_____ cm	_____ /_____ cm	_____ /_____ cm
Circumference	_____ cm	_____ cm	_____ cm
Volume	_____ **cc**	_____ **cc**	_____ **cc**

The volume of a typical marble in the jar is: _____cc

JAR: Circumference _____ cm Radius _____ cm Height _____ cm Volume _____ cc

SPACE: Volume of space in the jar: _____ cc

How many marbles do you ESTIMATE are in the jar? _____

How many marbles are actually in the jar? _____

What was your margin of Error? _____%