Road Rage MONSTER CAR SYSTEMS

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Objective

The objective for this lesson is three-fold: 1) To have students experience the application of solving systems of equations, 2) to understand the meaning of the graph of a system, adn 3) to offer the students more practical experience in writing linear equations.

Concepts
Solving systems in slope-intercept form; graphing systems; writing equations

Time: 2-3 hours

Materials
Two battery-operated cars of different speeds, stop watches, student handout, graph paper, calculators.

Preparation
Find the venue for the lesson and establish the "race course" as is done for the original Monster Cars lesson.

http://www.mathprojects.com/lessons.asp

Facilitating the Race

Rather than have each group conduct its own race (as was done with the original Monster Cars lesson), this time, the teacher and one other person conduct the race with only two cars, while student-volunteers time. In order to expedite the lessons, in each successive part (Parts 2-5), the teacher can calculate the rates of the cars beforehand and announce the rates to the class. If the rates of the two cars are too similar, one can be slowed done by placing extra weight on the car, or by using older batteries.

Part One: Slower Car Gets a Lead

a) Experience shows that this section, calculating rate, will be the hardest part of the lesson for students. They will have the distance and the time for each car. Once they have wrestled with how to calculate the rate, if necessary, hint to them that since the rate is in inches per second they should divide inches by seconds. Have students round the rates to the nearest tenth.

b) The starting point of Slower Car is determined arbitrarily for the most part, but the teacher does want to keep the race reasonable so the larger the discrepancy in the rates of the two cars, the larger the lead Slower Car should receive. Announce the starting point of the car, and be sure that students write the equations properly before you have them solve. A car with a rate of 9.4 seconds and a starting point that is 20 inches in front of the starting line will have an equation of $d = 9.4t + 20$.

Solving the system will actually be one of the easiest steps in the lesson. The emphasis here should be on what that solution means in terms of the race. When the class returns to the racecourse, mark the starting point of Slower Car and the projected point at which Faster Car catches Slower Car (front bumpers are equal distance from the starting line. Though it will be very unlikely for the actual time and distance to equal the predicted time, record them anyway. You may be amazed at how close the student calculations really are.

c) Graphing poses some unique challenges in this lesson. The lesson is light on data points, and slopes such as 9.4 are difficult to count on one-quadrant graphs, which have different scales on the two axes. It is best to have the students plot the y-intercepts and chose one other point. For example, the students could evaluate the equations for a time of 5 and plot those points. With a data point and y-intercept for each equation the students could then easily graph the system.

Have the students focus on the relationship between their algebraic solution from the previous question and the point of intersection on the graph.

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PART TWO: BOTH CARS GET A LEAD
The process is identical to part one except that both cars start in front of the starting line, so both equations have a y-intercept other than zero.

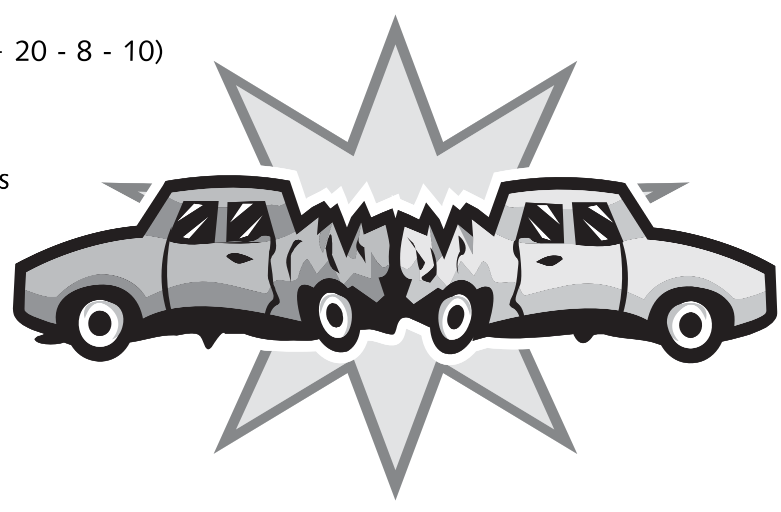
PART THREE: COLLISION COURSE
This is the fun one!! The mathematical process is again identical to the first two parts, with the exception that one car has a negative rate, and the other has a negative y-intercept. The interest peaks for the students due to the predicted collision factor.

These next two sections are extensions of this lesson. They can be used for differentiation or extra credit.

PART FOUR: T-BONE CRASH
The mathematics for this exercise are quite simple, but it is the conceptual piece that is challenging for the students. They are given the rate of both cars and the distance between the two x's on the ground. It is up to them to calculate the distance that Faster Car should start from the point of collision, so that the two cars actually do collide on the "x." They, of course, do this by first calculating the time it should take Slower Car to drive from one mark to the other, and then use that time (needs to be the same for both cars in order to guarantee a collision) to calculate the needed distance.

PART FIVE: THE TRAILER HITCH
This is a rather advanced problem for Algebra One, but most appropriate for Algebra 2 students. Since the solutions here are a range of times and distances, the algebraic sentence and its accompanying graph involve inequalities. Let us assume that the cars are each eight inches long, the trailing string is 10 inches long, Faster Car starts 15 inches behind the starting line with a rate of 18 in/sec, and Slower Car begins 20 inches in front of the starting line with a rate of 11 in/sec. The equations representing this scenario are as follows:

Front of Slower Car: $d = 11t + 20$
Back of String: $d = 11t + 2$ $(11t + 20 - 8 - 10)$
Faster Car: $d = 18t - 8$



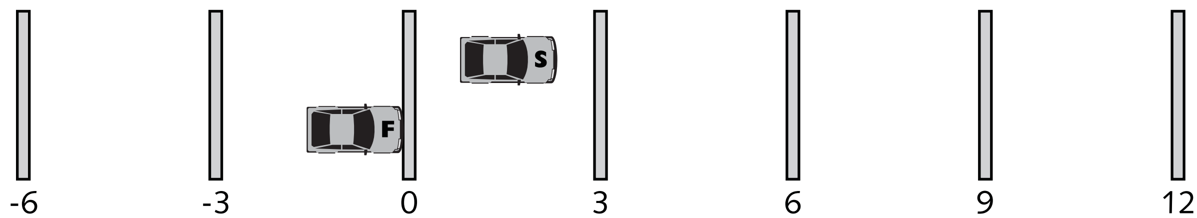
These equations may be combined to represent the infinite numbers of solutions (times and distances) as follows:

$$11t + 2 < 18t - 8 < 11t + 20$$

The solution here, $1.4 < t < 4$, means that the car will reach the string at 1.4 seconds, and pass the front end of the other car at 4 seconds.

Road Rage MONSTER CAR SYSTEMS

You have two toy cars. The two cars have different speeds, so they will be referred to as Faster Car and Slower Car. Using their respective rates and starting points, calculate when the two cars will be the same distance from the starting line. Each mark is 3 feet apart as shown in the diagram below.



PART ONE: SLOWER CAR GETS A LEAD

a. Determine the rate of both cars by timing how long it takes each to go from the starting line to the first mark.

	Time to the First Mark	Rate (in/sec)
Faster Car:	____________________	____________________
Slower Car:	____________________	____________________

b. Place Faster Car at the starting line. Place Slower Car at its predetermined starting point somewhere between the first and second marks. Calculate when Faster Car will pass Slower Car, and at what distance.

1) Starting Point of Slower Car: _______________

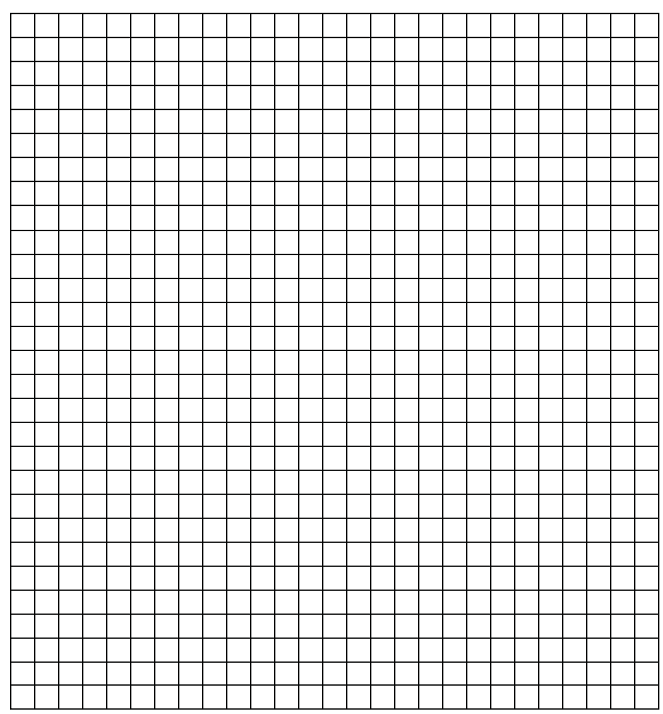
2) Equation of Faster Car: _______________

3) Equation of Slower Car: _______________

4) Predicted time and distance: (,)

5) Actual time and distance: (,)

c. Graph this system.



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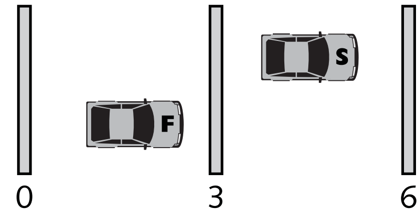
PART TWO: BOTH CARS GET A LEAD



Place Faster Car at its predetermined starting point somewhere between the first and second marks. Place Slower Car at its predetermined starting point somewhere between the second and third marks.

a) Write a system of equations that represents the time and distance of the two cars:

$\Bigg\{$

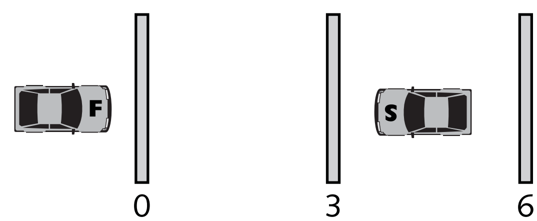
b) Use the system above to calculate when Faster Car will pass Slower Car, and at what distance.

Predicted time and distance: (,)

Actual time and distance: (,)

c) Graph the system.

PART THREE: COLLISION COURSE



Place Faster Car at its predetermined starting point somewhere behind the starting line. Place Slower Car at its predetermined starting point somewhere between the second and third marks facing the starting line.

a) Write a system of equations that represents the time and distance of the two cars:

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b) Use the system above to calculate when the two cars will collide, and at what distance.

Predicted time and distance: (,)

Actual time and distance: (,)

c) Graph the system.

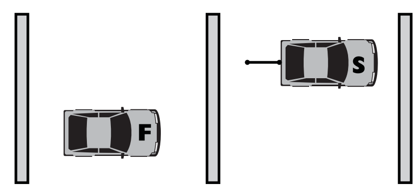
Road Rage: MONSTER CARS SYSTEMS *(continued)*

PART FOUR: T-BONE CRASH

Slower Car has gone renegade and is driving perpendicular across the racecourse. Place Slower Car at the first "X." Assuming Faster Car is traveling the racecourse in a normal direction, determine the starting point of Faster Car so that the two cars collide at the second "X."

	Rate (in/sec)	Distance to X
Faster Car:	____________________	____________________
Slower Car:	____________________	____________________

PART FIVE: THE TRAILER HITCH



Slower Car has a 10 inch string attached to its bumper that it drags along the ground. Place Slower Car at its predetermined starting point and place Faster Car somewhere behind Slower Car. Determine when the front end of Faster Car will be between the tail end of the string and the front end of Slower Car.

a) Equations for:

Front of Slower Car: ____________________

Back of String: ____________________

Faster Car: ____________________

b) Inequality: ____________________

c) Solution: ____________________

d) Graph the three equations.
How does the graph relate to the solution of the compound inequality?