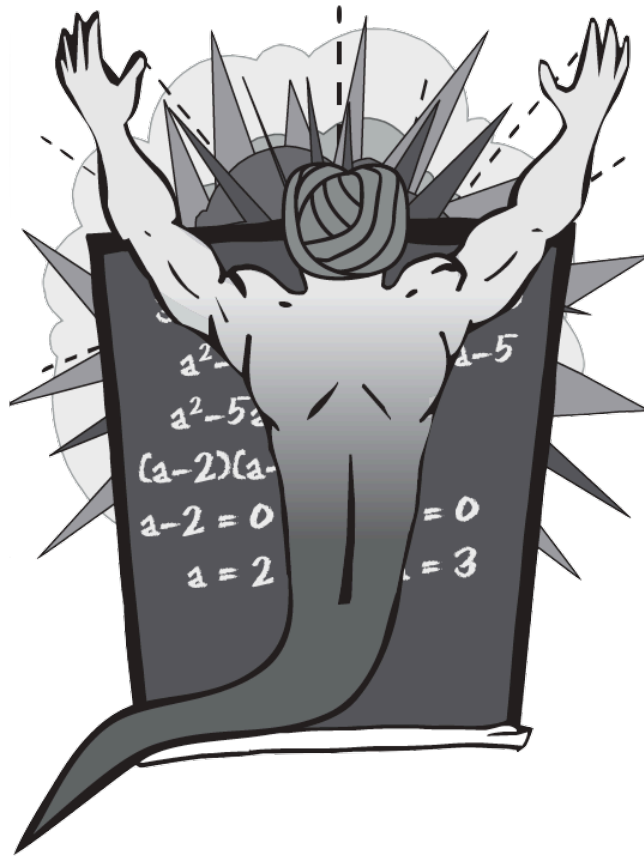


# Ultimate Cosmic Power:

## In An Itty-Bitty Thinking Space

Algebraic Reasoning for OK-MAP, June 2017



**Chris Shore**

*The Math Projects Journal*

shore@mathprojects.com

@MathProjects

#OKMAP2017

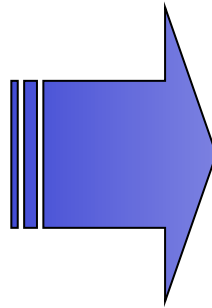


# What is Your Million-Dollar Talent?



# The Mindset Shift

Fixed  
Mindset



Growth  
Mindset ...

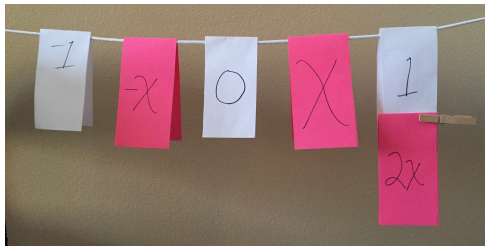
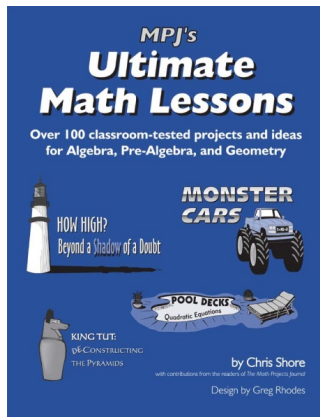


*They are that smart  
&  
We are that good!*



... of the teachers!

# The World That I Come From

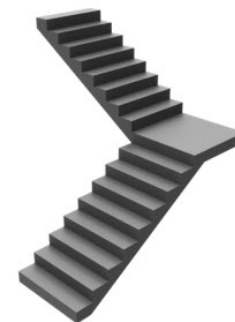




# Today's Ultimate Cosmic Power

**Define**  
**Algebraic Reasoning**

WHY?



HOW?

**Learn the Key to Teaching**  
**Algebraic Reasoning**



WHAT?

**Engage in Lessons for**  
**Algebraic Reasoning**



# What is Algebraic Reasoning?

according to the uninitiated...

---

“Algebra would be a lot easier if they just told you what  $x$  was.”  
-- Scott, Class of '94

“Only in math do you put two things together and get a smaller thing.”  
-- Neal, Class of '99

“You math teachers aren't very good. My whole life you have been asking people to find  $x$ . Why can't you find it yourselves?”  
-- Angry English Teacher

# What is Algebraic Reasoning?

according to you...

---



[peardeck.com/join](https://peardeck.com/join)

Code:



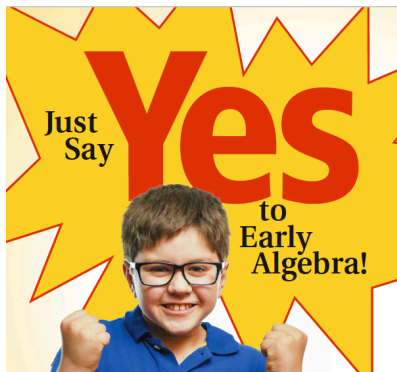
# What is Algebraic Reasoning?

according to a disgruntled math teacher...

---

“ $x + 3 = 5$  is NOT algebra.”

Twenty students are in your class. How many boys, how many girls? What other combinations of boys and girls can there be in a class of twenty students? Using the variables  $b$  for the number of boys and  $g$  for the number of girls, how can we represent all possible combinations of boys and girls?



$$b + g = 20$$

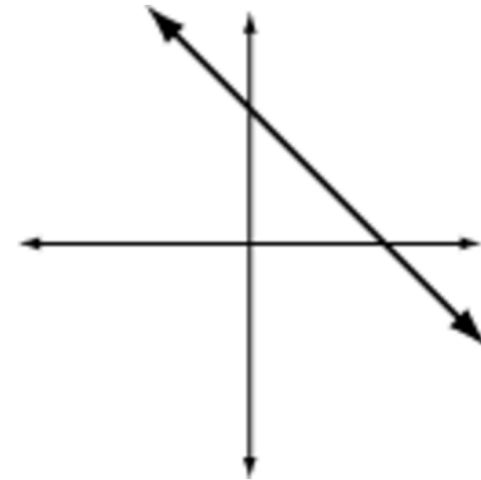


# What is Algebraic Reasoning?

according to me (and the ancients) ...



$$x + y = 7$$



# What is Algebraic Reasoning?

according to your Oklahoma State Standards ...

---

... algebra is more than moving symbols around.

It is about ...

*understanding **patterns, relations and functions**,  
representing and analyzing mathematical  
situations and structures using algebraic **symbols**,  
using mathematical **models** to represent and  
**understanding quantitative relationships**, and  
analyzing **change** in various contexts.*



# What is Algebraic Reasoning?

## according to the Progression ...

Algebraic Reasoning & Algebra (A)		
Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)

# What is Algebraic Reasoning?

According to the State's Vision & Guiding Principles...

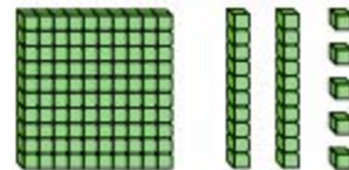
... a strong mathematics program that emphasizes and engages [mathematically proficient and literate students] in problem solving, communicating, reasoning and proof, making connections and using representations.

Equity

Curiosity,  
Enjoyment &  
Understanding

Problem Solving

Technology





# What is Algebraic Reasoning?

## According to the Mathematical Actions & Processes...

*Throughout their Pk-12 education experience, mathematically literate students will:*

### **Develop a Deep and Flexible Conceptual Understanding**

Demonstrate a deep and flexible conceptual understanding of mathematical concepts, operations, and relations while making mathematical and real-world connections. Students will develop an understanding of how and when to apply and use the mathematics they know to solve problems.

### **Develop Accurate and Appropriate Procedural Fluency**

Learn efficient procedures and algorithms for computations and repeated processes based on a strong sense of numbers. Develop fluency in addition, subtraction, multiplication, and division of numbers and expressions. Students will generate a sophisticated understanding of the development and application of algorithms and procedures.

### **Develop Strategies for Problem Solving**

Analyze the parts of complex mathematical tasks and identify entry points to begin the search for a solution. Students will select from a variety of problem solving strategies and use corresponding multiple representations (verbal, physical, symbolic, pictorial, graphical, tabular) when appropriate. They will pursue solutions to various tasks from real-world situations and applications that are often interdisciplinary in nature. They will find methods to verify their answers in context and will always question the reasonableness of solutions.

### **Develop Mathematical Reasoning**

Explore and communicate a variety of reasoning strategies to think through problems. Students will apply their logic to critique the thinking and strategies of others to develop and evaluate mathematical arguments, including making arguments and counterarguments and making connections to other contexts.

### **Develop a Productive Mathematical Disposition**

Hold the belief that mathematics is sensible, useful and worthwhile. Students will develop the habit of looking for and making use of patterns and mathematical structures. They will persevere and become resilient, effective problem solvers.

### **Develop the Ability to Make Conjectures, Model, and Generalize**

Make predictions and conjectures and draw conclusions throughout the problem solving process based on patterns and the repeated structures in mathematics. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.

### **Develop the Ability to Communicate Mathematically**

Students will discuss, write, read, interpret and translate ideas and concepts mathematically. As they progress, students' ability to communicate mathematically will include their increased use of mathematical language and terms and analysis of mathematical definitions.

# What is Algebraic Reasoning?

---

## Why Teach It?



# Teaching Algebraic Reasoning Through Explicit Instruction

H.O.T.S.

Dr. John Star



“Math does not teach Problem Solving.”

“Only the explicit teaching of thinking  
teaches thinking.”



# Teaching Algebraic Reasoning Through Problem Solving

---

**Exercise**



**Problem**



**Don't Know How**  
**Have the Ability**

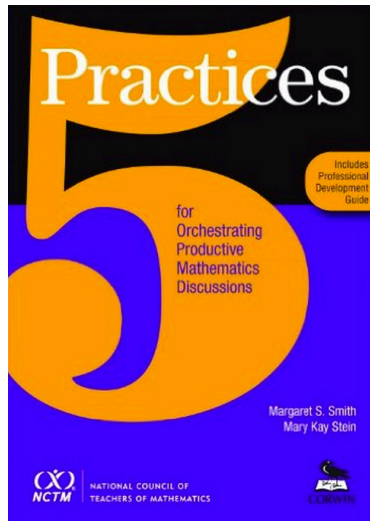


**Crisis**





# Teaching Algebraic Reasoning Through Tasks



Dr. Peg Smith

“It’s all about the task.  
It’s all about the task.  
It’s all about the task.”

# Teaching Algebraic Reasoning Through Tasks

---

“a problem that provides an opportunity to develop mathematical ideas and [thinking].”

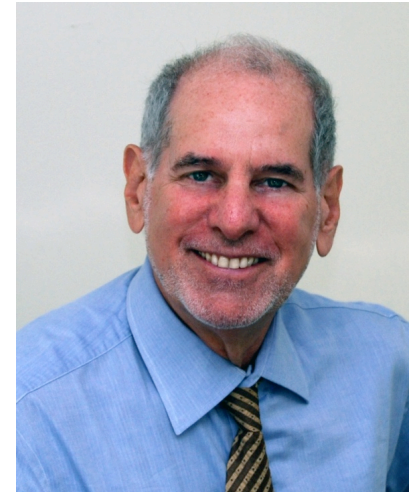
-- Adding It Up (2001)

Tasks = Problems used to teach Content & Processes



# Teaching Algebraic Reasoning To Whom?

“Accelerated” Remedial Math Students  
with Rich & Robust Tasks



Dr. Uri Treisman

**ALL Kids!**

Dave Foster

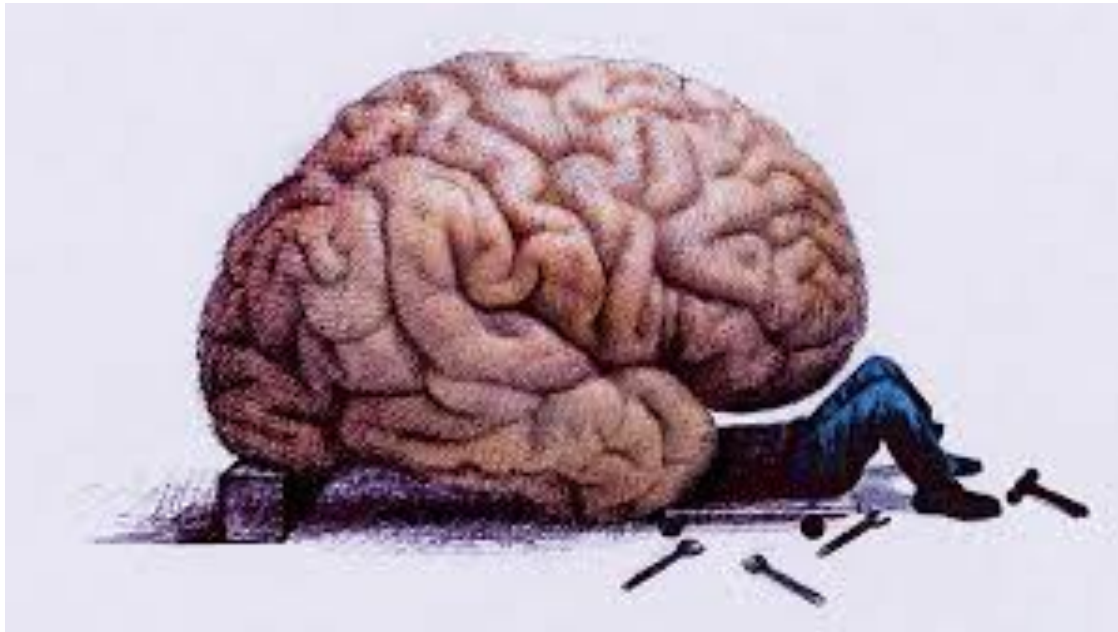


50% False Positives  
Among 8<sup>th</sup> Grade Geometry  
From CST to SBAC

# Teaching Algebraic Reasoning

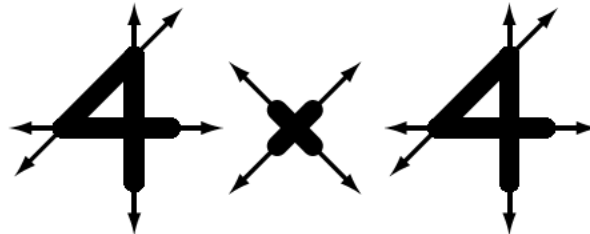
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How Is It Best Taught?

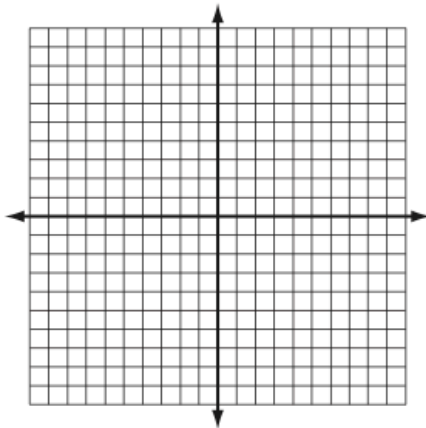




# Lessons for Algebraic Reasoning



1)



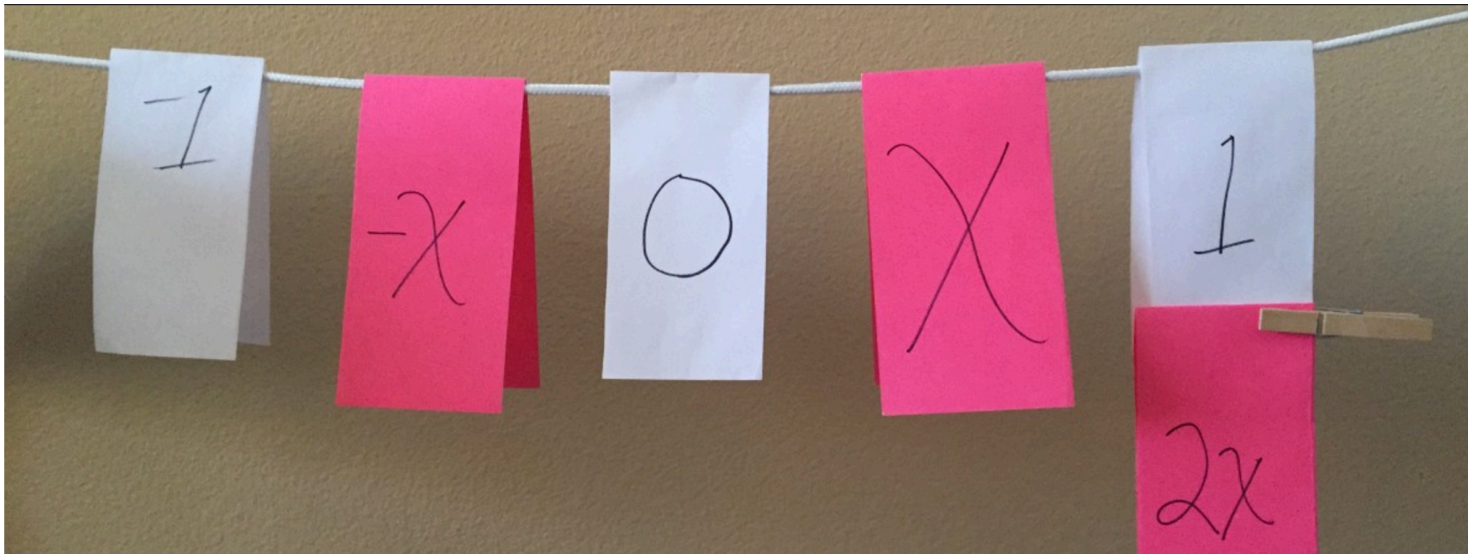
Equation:

\_\_\_\_\_

Scenario: Fred moves into town with no friends, and makes one new friend every day.

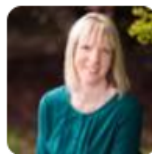
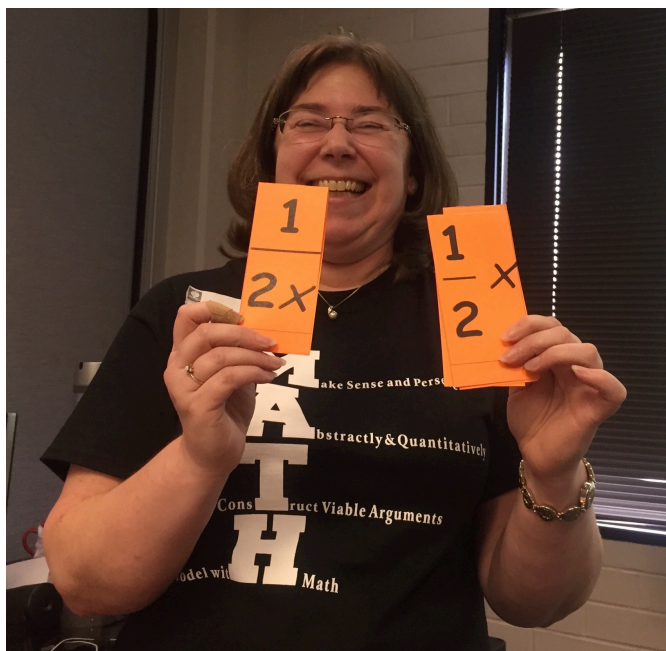
# Lessons for Algebraic Reasoning

## *The Master Number Sense Maker!* Clothesline Math



# Clothesline Math

## A Brief History



**Molly Daley**

@mdaley15

I've just been Clotheslined or maybe kicked in the head.

[@MathProjects](#) #55thNWMC

# Clothesline Math

## Exponential Decay or Order of Ops?

$$y = 6\left(\frac{1}{2}\right)^x$$

$6\left(\frac{1}{2}\right)^0$	$6\left(\frac{1}{2}\right)^1$	$6\left(\frac{1}{2}\right)^2$
-------------------------------	-------------------------------	-------------------------------

# Clothesline Math Handout



Name: \_\_\_\_\_  
Date: \_\_\_\_\_



For each set, record the given values, expressions or drawings. After the discussion of their placement on the clothesline, record them on the number line.

1. \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_



**Discussions, Deductions & Decisions**



# Clothesline Statistics

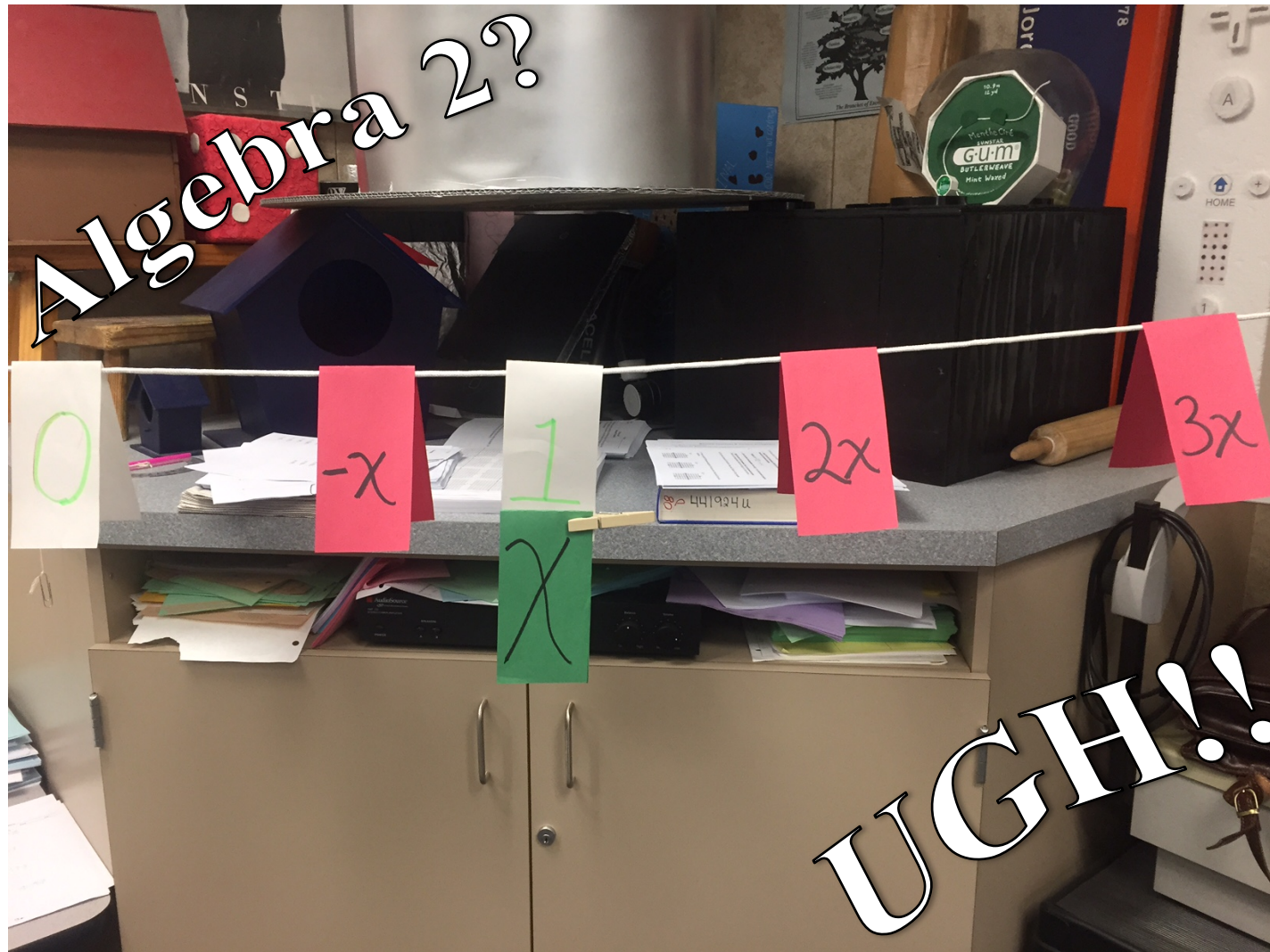
## *Logarithms*

---

$\log_4 16$	$\log_2 \frac{1}{8}$	$\ln 1$
-------------	----------------------	---------



# Clothesline



# Clothesline Math as Review



$$\sqrt[3]{8}$$

$$25^{\frac{1}{2}}$$

$$-2.5$$

$$5^{-2}$$

$$(-2)^0$$

$$\frac{3}{2}$$

$$\sqrt{2}$$

$$73\%$$

$$16^{\frac{1}{4}} + 32^{\frac{1}{5}}$$

$$4^{\frac{3}{2}}$$

$$-0.08$$

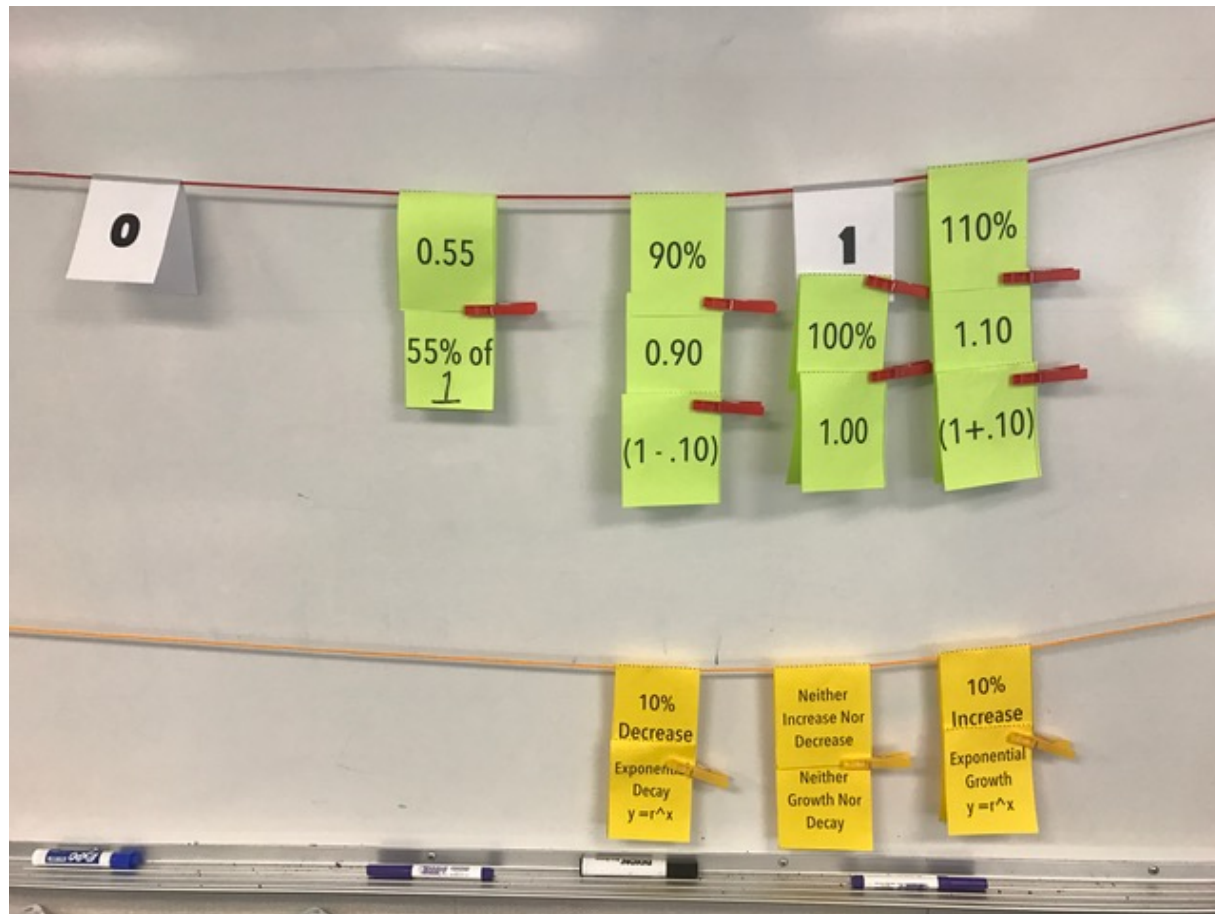
$$-\sqrt{3}$$

$$8^{-\frac{1}{3}}$$

$$\sqrt{42}$$

# Clothesline Math

## on conceptual warm-ups





# www.clotheslinemath.com



## Clothesline Math

The Master Number Sense Maker

[Home](#)

[Blog](#)

[Making the Clothesline](#)

[Benchmarks](#)

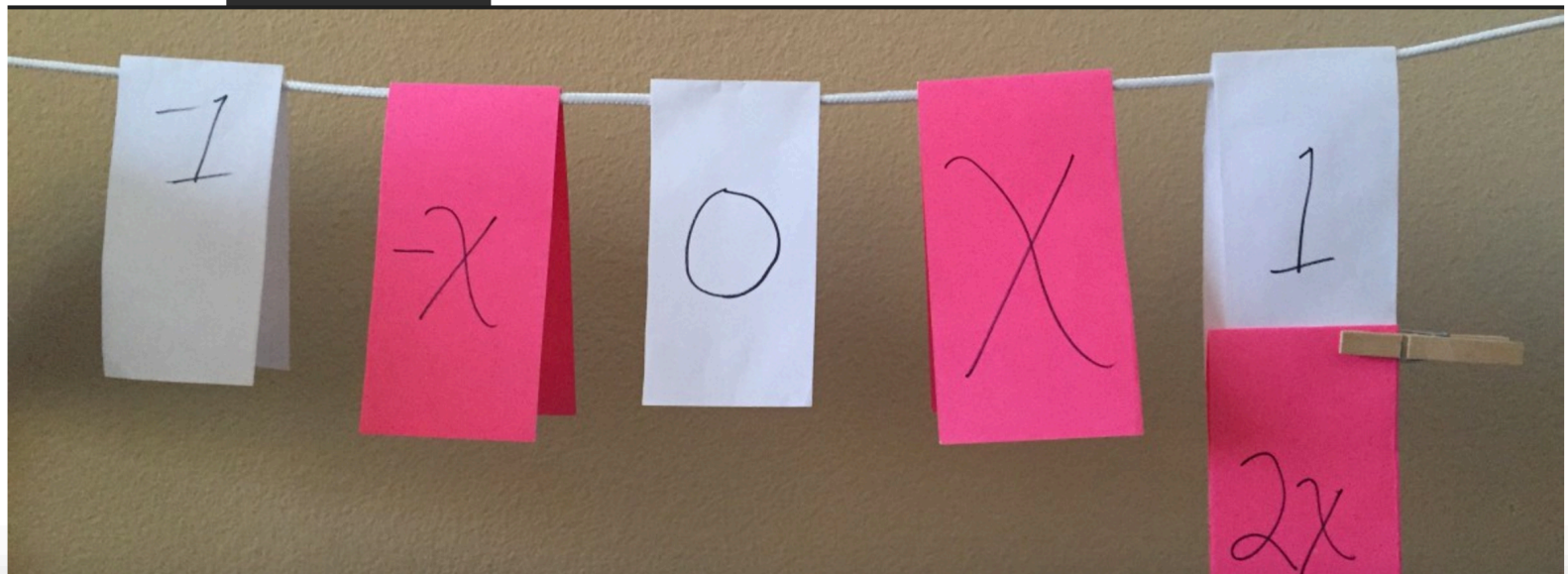
[Numbers](#)

[Algebra](#)

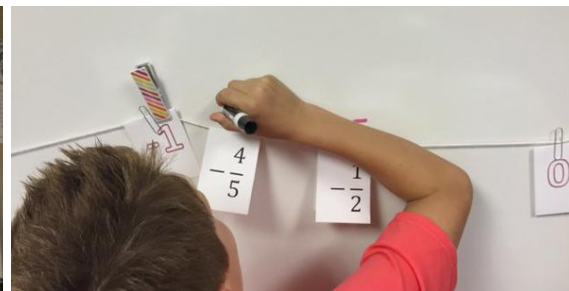
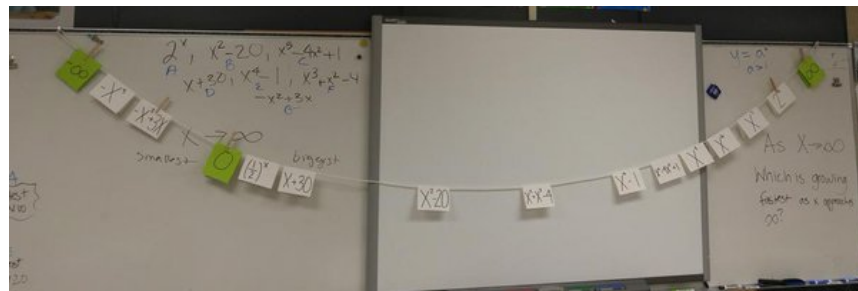
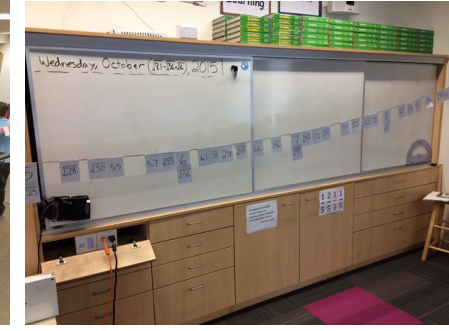
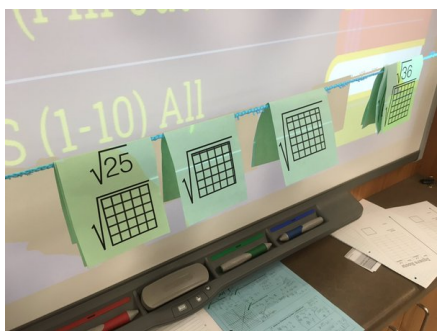
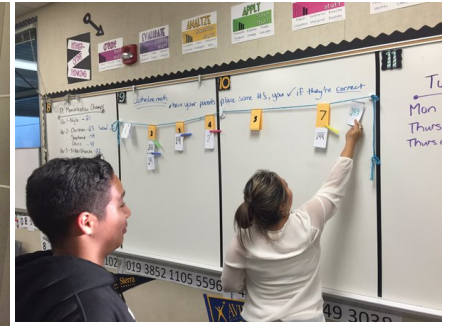
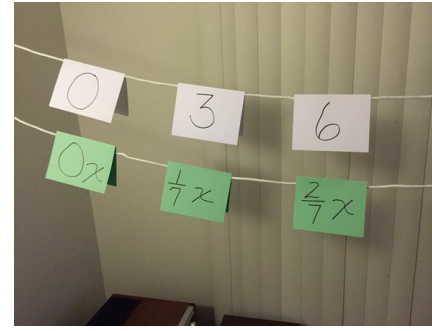
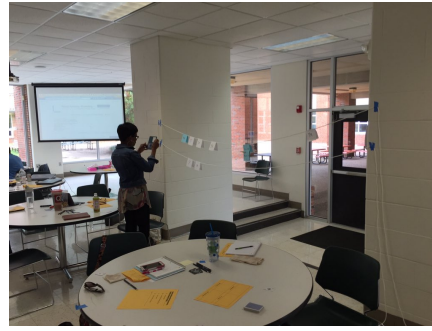
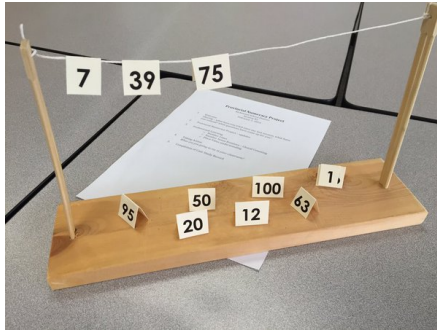
[Functions \(graphs\)](#)

[Geometry](#)

[Statistics](#)

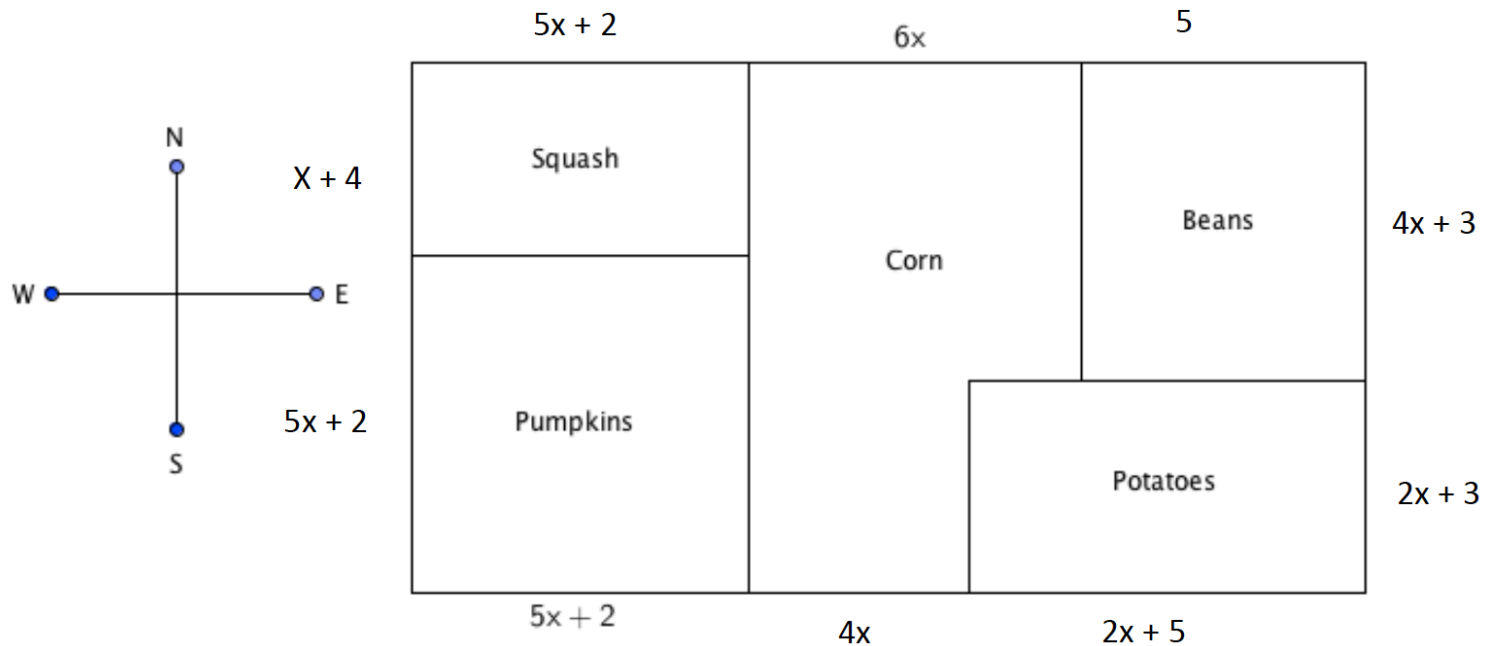


# Clothesline Math Around the Nation



# Lessons for Algebraic Reasoning

## Polynomial Farm





Day 1 & 2

# Lessons for Algebraic Reasoning

**Conceptual → Procedural → Application  
Practice**

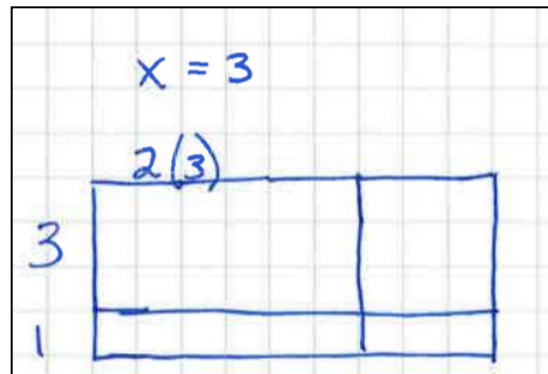
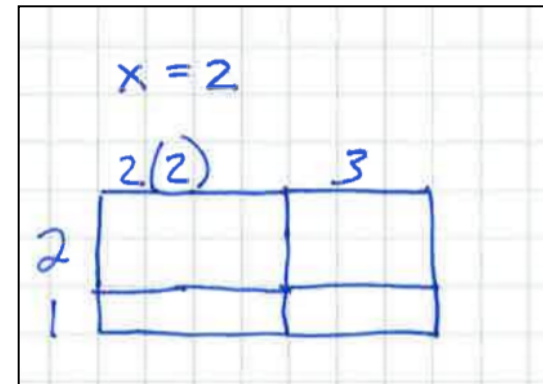
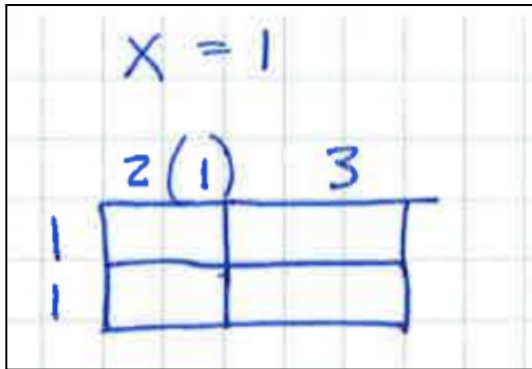
	4in.	5in.
2in.	A	B
3in.	C	D

	2x	3
x	A	B
1	C	D

	Perimeter	Area
A		
B		
C		
D		
Large Rectangle		

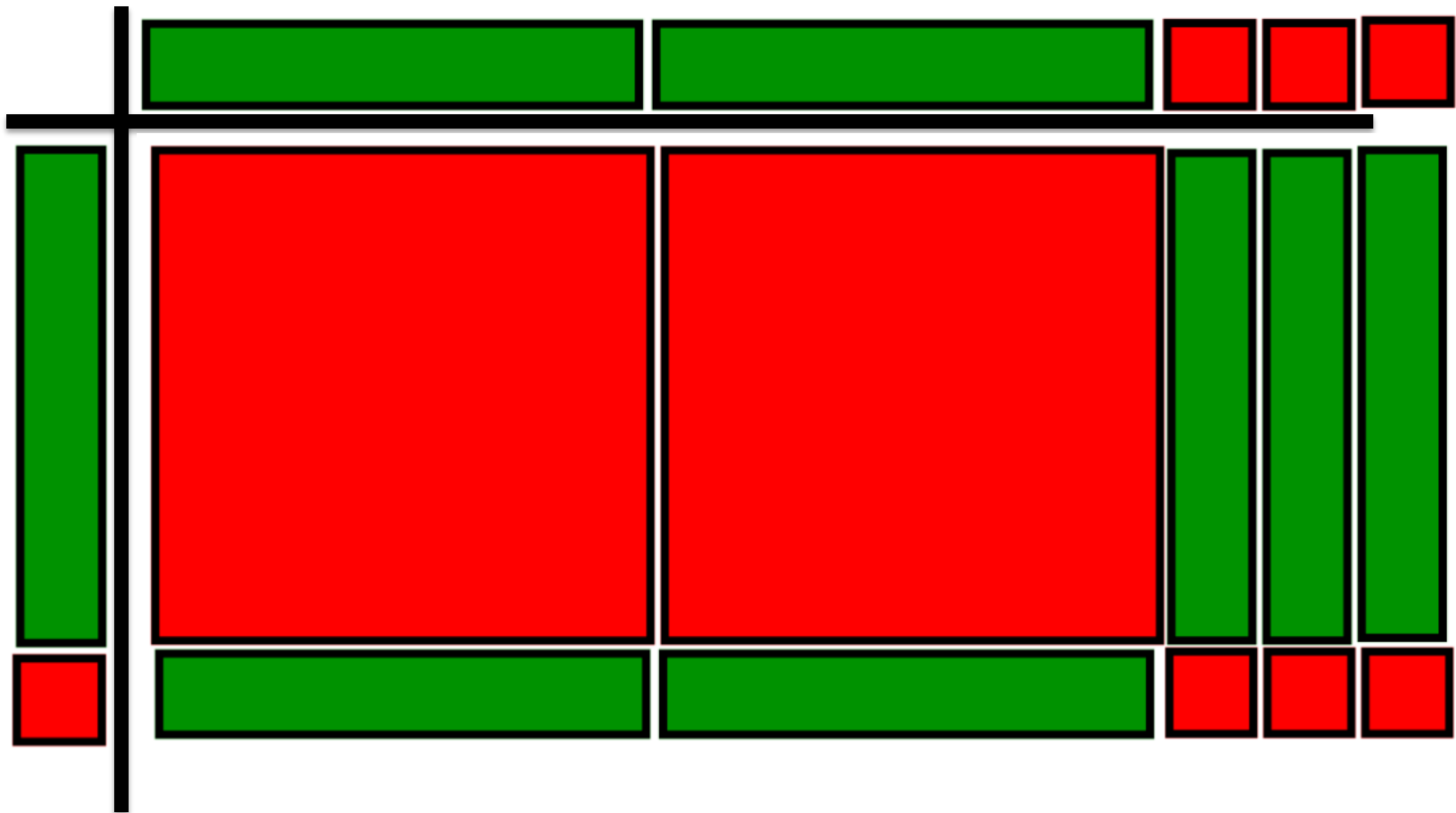
Day 3

# Lessons for Algebraic Reasoning



Day 4

# Lessons for Algebraic Reasoning

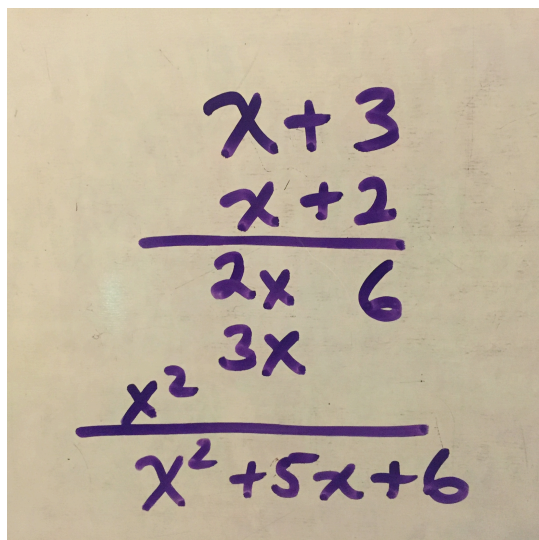


Day 6

# Lessons for Algebraic Reasoning

Conceptual → Procedural → Application Practice

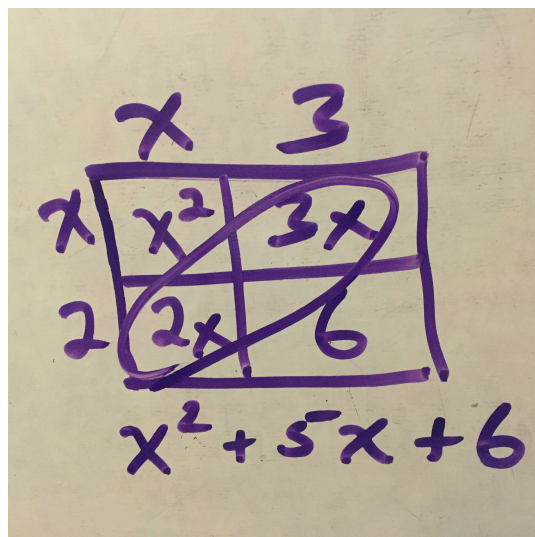
Stacking Method



Handwritten work showing the stacking method for multiplying  $(x+3)(x+2)$ . The first polynomial  $x+3$  is written above the second  $x+2$ . A horizontal line separates them. Below the line, the products are stacked:  $2x$  and  $6$  (from  $2 \times (x+3)$ ), then  $x^2$  and  $3x$  (from  $x \times (x+3)$ ). A final horizontal line is drawn, and the sum  $x^2 + 5x + 6$  is written below.

$$\begin{array}{r} x+3 \\ \times x+2 \\ \hline 2x \quad 6 \\ x^2 \quad 3x \\ \hline x^2 + 5x + 6 \end{array}$$

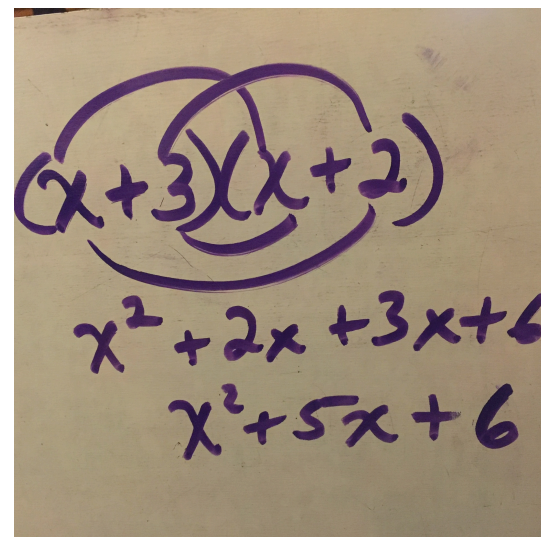
Area Model  
Box Method



Handwritten work showing the area model (box method) for multiplying  $(x+3)(x+2)$ . A rectangle is divided into six smaller boxes by a vertical line and a horizontal line. The top-left box is labeled  $x^2$ , the top-right  $3x$ , the bottom-left  $2x$ , and the bottom-right  $6$ . The top edge is labeled  $x$  and  $3$ , and the left edge is labeled  $x$  and  $2$ . A diagonal line is drawn from the top-left to the bottom-right. Below the box, the sum  $x^2 + 5x + 6$  is written.

$$\begin{array}{|c|c|} \hline x & 3 \\ \hline x & x^2 & 3x \\ \hline 2 & 2x & 6 \\ \hline \end{array}$$
$$x^2 + 5x + 6$$

FOIL method

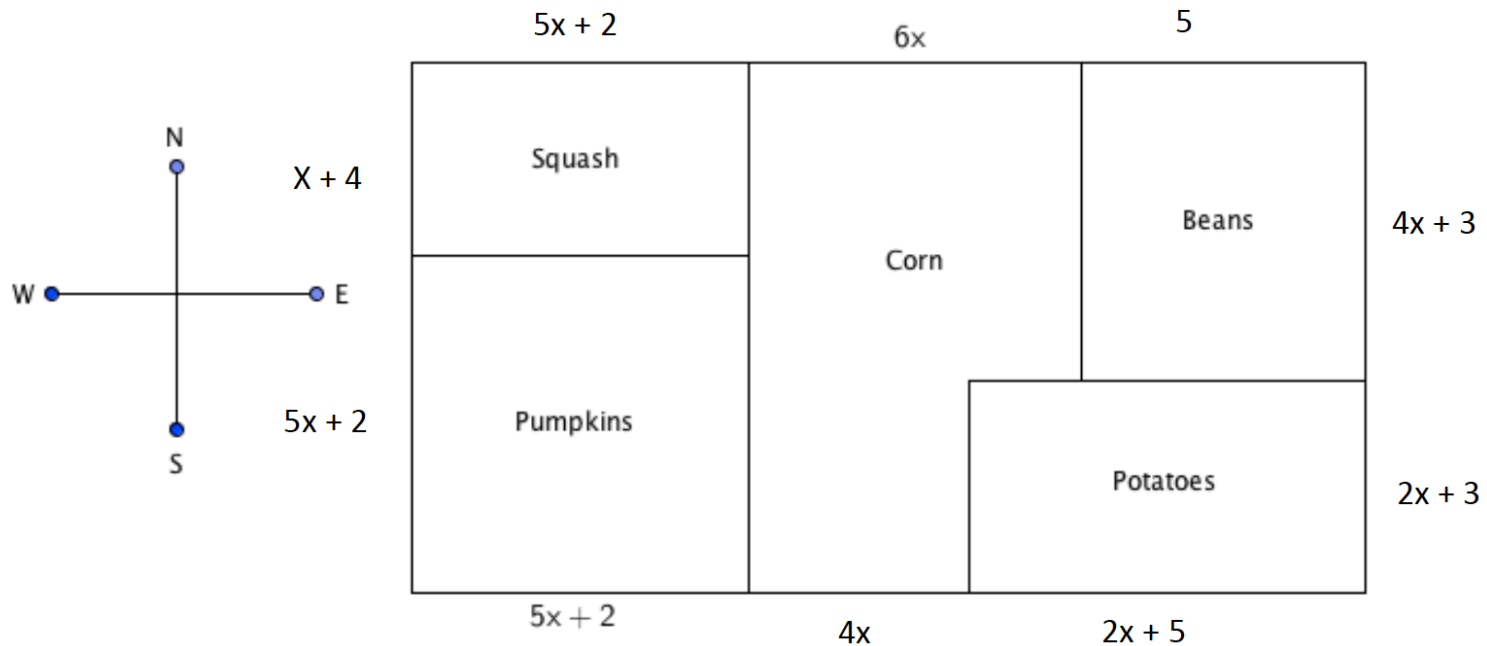


Handwritten work showing the FOIL method for multiplying  $(x+3)(x+2)$ . The expression  $(x+3)(x+2)$  is written and circled. Below it, the terms are listed:  $x^2 + 2x + 3x + 6$ , followed by the simplified sum  $x^2 + 5x + 6$ .

$$(x+3)(x+2)$$
$$x^2 + 2x + 3x + 6$$
$$x^2 + 5x + 6$$

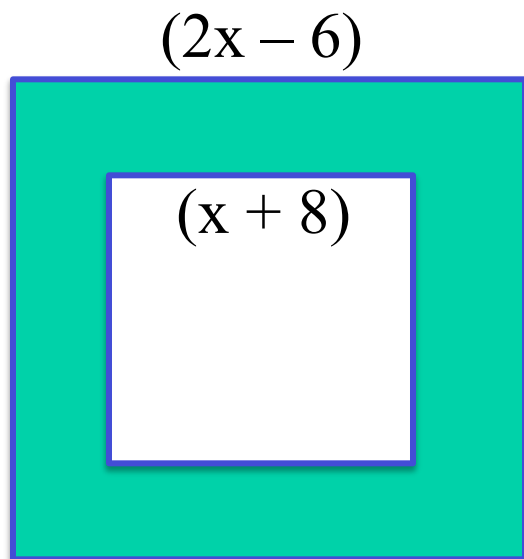
Day 7

# Lessons for Algebraic Reasoning



# Lessons for Algebraic Reasoning

Audra is framing a square painting with side lengths of  $(x + 8)$  inches. The total area of the painting and the frame has a side length of  $(2x - 6)$ . The material for the frame is \$0.10 per square inch.



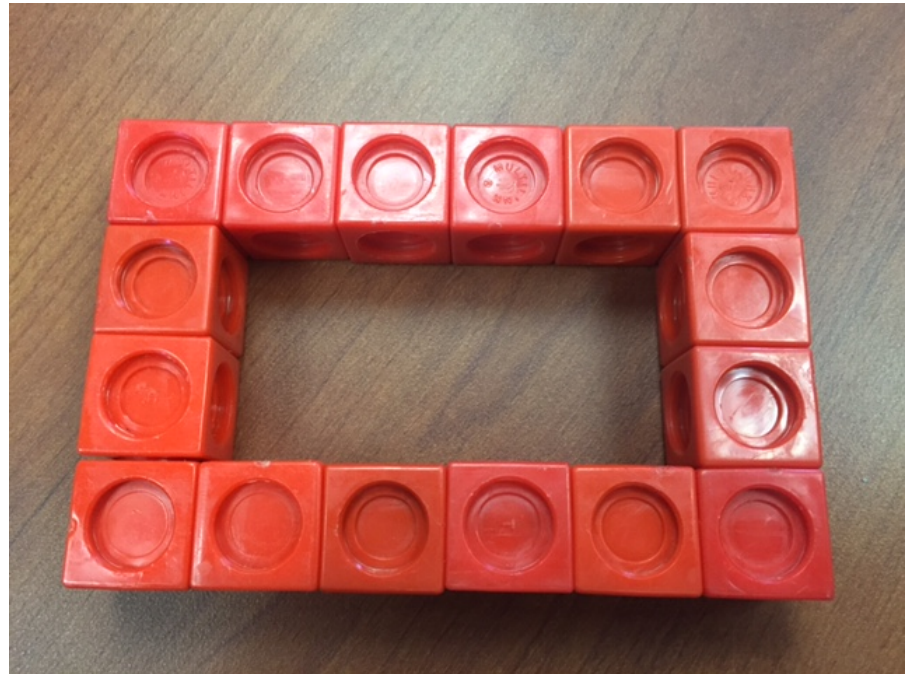
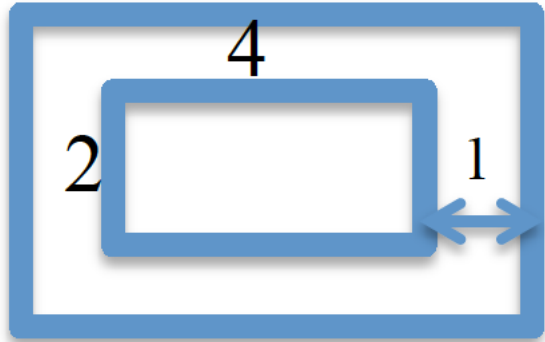
$$(2x - 6)^2 - (x + 8)^2$$

1. Write the expression for the area of the painting.
2. Write the expression for the area of the painting and the frame.
3. Write the expression for the area of the frame.
4. Find the area of the frame if  $x=16$ .
5. Find the cost of the material for the frame.

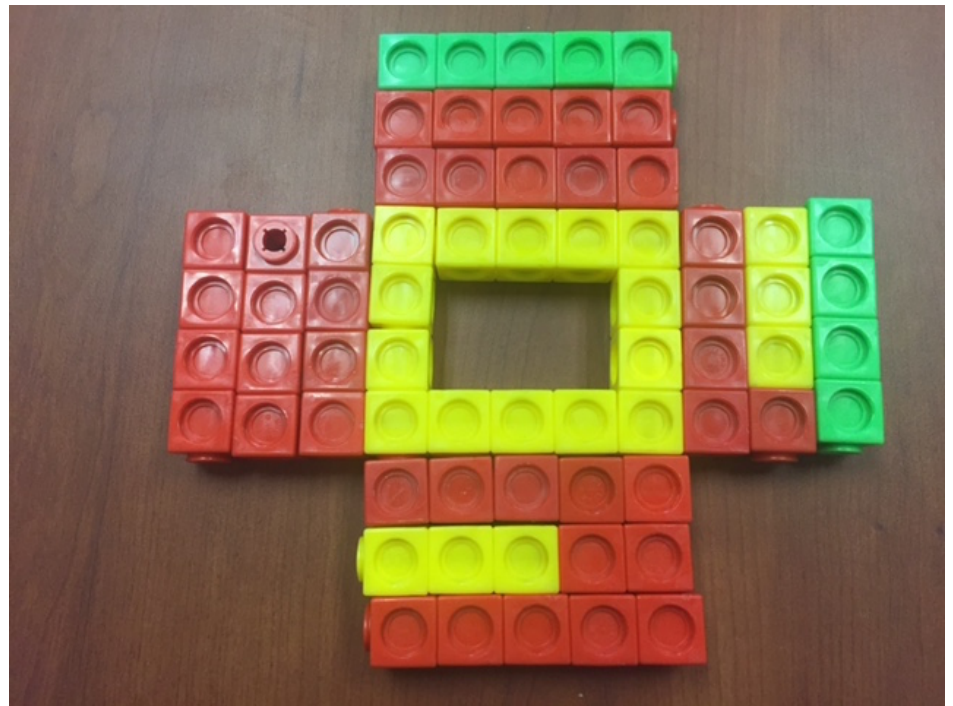
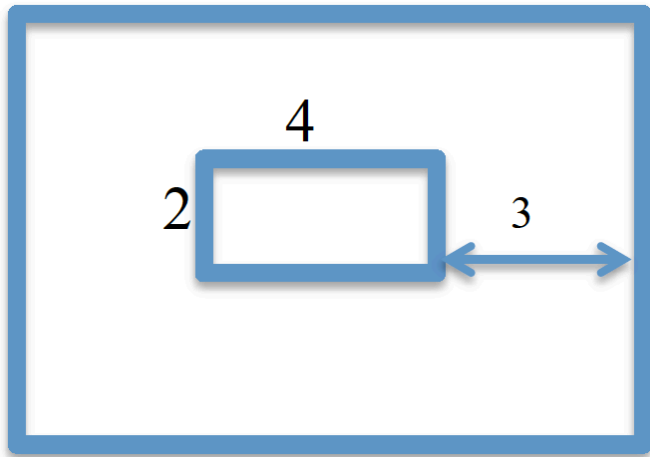


# Lessons for Algebraic Reasoning

Day 10-11



# Lessons for Algebraic Reasoning



# Lessons for Algebraic Reasoning

Conceptual → Procedural → Application Practice

	2x	3
x	A	B
1	C	D

Handwritten algebraic work showing the multiplication of two binomials:

$$(x+3)(x+2)$$
$$x^2 + 2x + 3x + 6$$
$$x^2 + 5x + 6$$

Diagram illustrating a difference of squares:

Outer square side length:  $(2x - 6)$

Inner square side length:  $(x + 8)$

Lowest Grade on District Benchmark for Polynomials = **76%**

Day 15

# Lessons for Algebraic Reasoning

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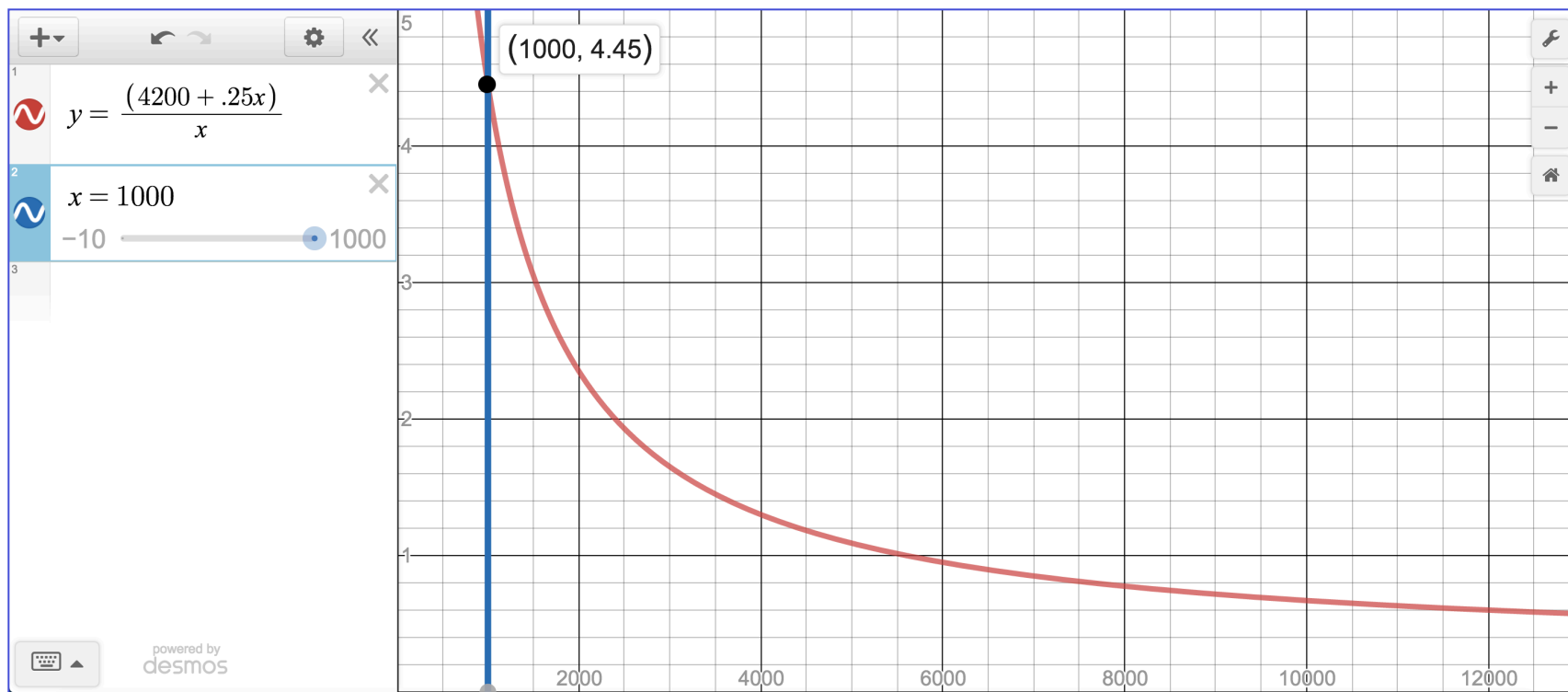
## Optimal Bait Company



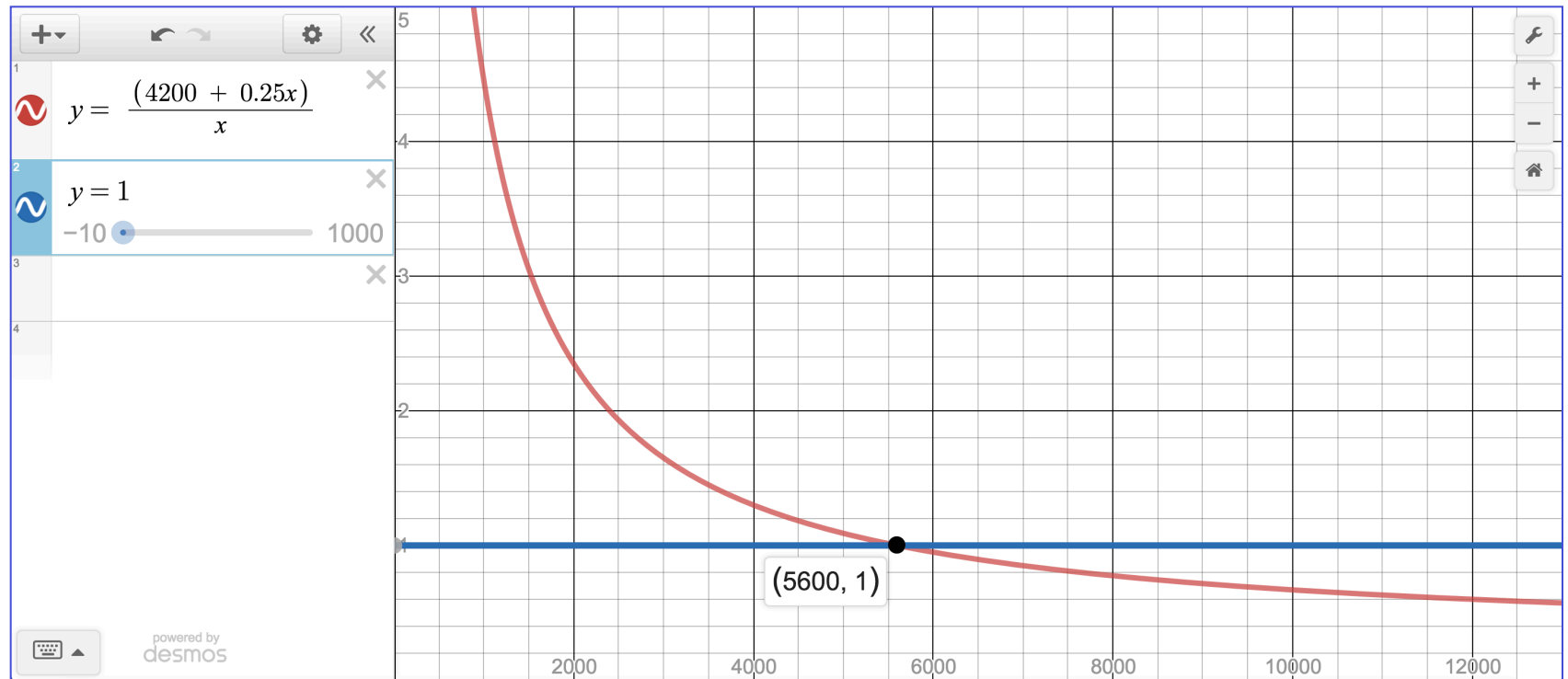
#1, 2 & 5



# Lessons for Algebraic Reasoning



# Lessons for Algebraic Reasoning





# What About From Your Textbooks?



# Teaching Algebraic Reasoning

---

What Lessons and Techniques  
Will You Use?



# Thoughts on Math

by the Initiated

---

“If you know how to do one problem inside and out, you can do a hundred just like it.”

-- Seheti, Math Teacher from India

“You just showed us several ways to do one problem, instead of one way to do several problems.”

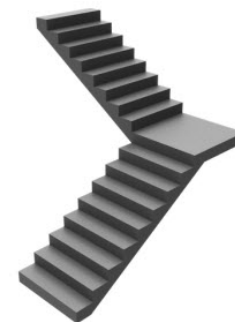
“Exactly!” -- Conversation with Dr. Tom Bennett,  
CSU San Marcos

“Poetry is the language of love. Math is the language of everything else.” -- Jon, Class of '99, U.S. Navy

# Ultimate Cosmic Targets

**Define**  
**Algebraic Reasoning**

WHY?



HOW?

**Learn the Key to Teaching**  
**Algebraic Reasoning**



WHAT?

**Engage in Lessons for**  
**Algebraic Reasoning**



# Call to Action

*Fail Grandly*

*No Real Risk*



10%

*2-Week Rule*



1 Thing

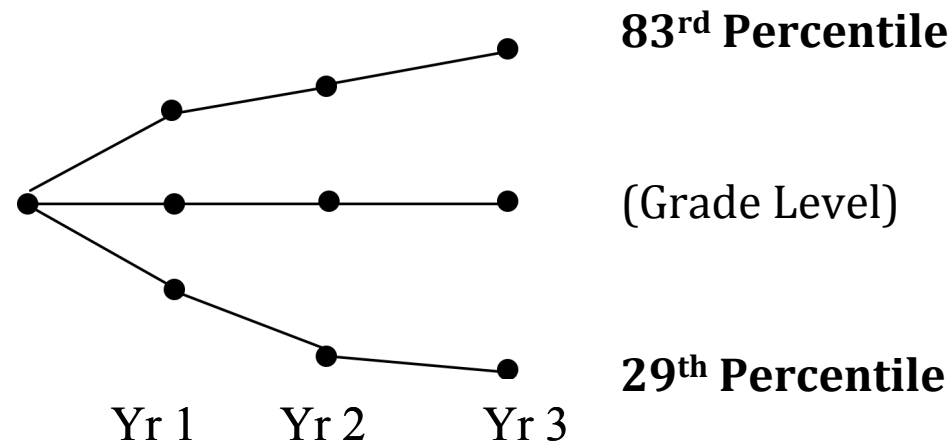




# Teacher Action is the Difference

**“The greatest influence in the quality of the education that a student receives is the decisions that a teacher makes on a daily basis.”**

-- Dr. William Schmidt, University of Michigan



# Teacher Action is the Difference



**Teachers matter most.**

-- Dr. William Schmidt, University of Michigan

**Teachers matter most.**

-- Peg Smith, University of Pittsburgh

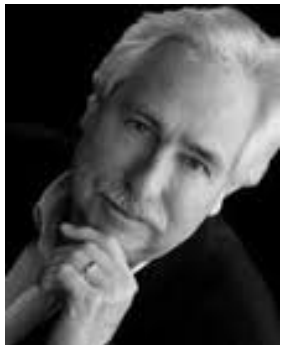


**Teachers matter most.**

-- Tim Kanold, Adlai E. Stevenson HS, Chicago

**Poverty matters ... a lot.**

-- Dr. Uri Treisman, University of Texas, Austin



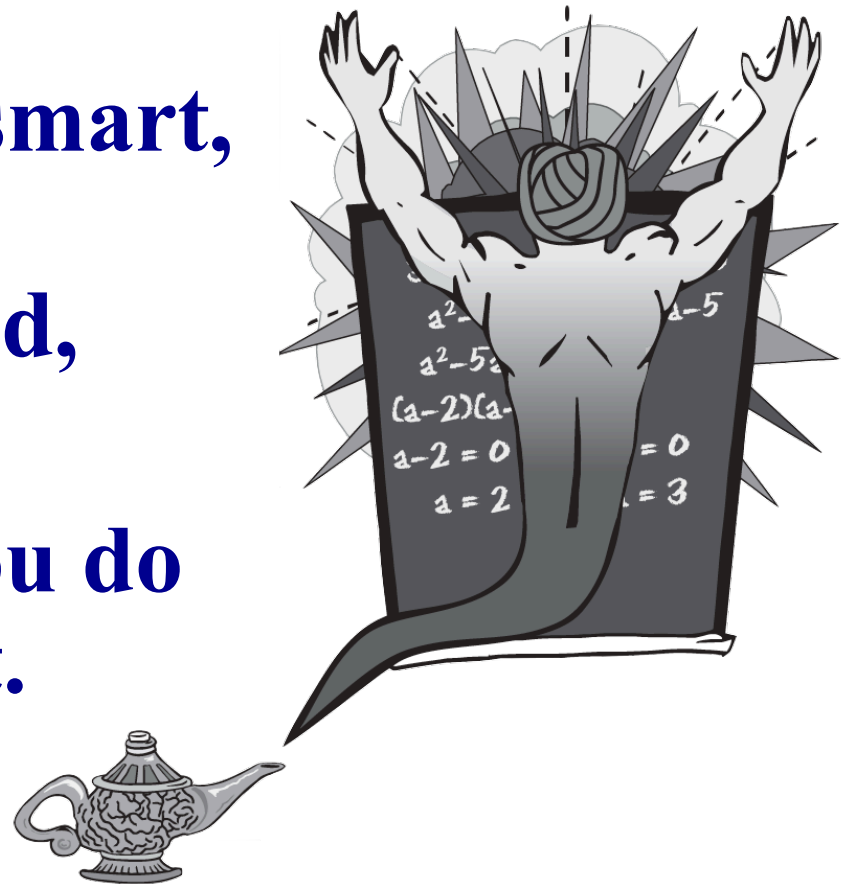
**Teachers matter most.**

-- David Foster, Silicon Valley Math Initiative, CA



# Instill Ultimate Cosmic Power in your students ...

because they are that smart,  
and  
we are that good,  
  
and because what you do  
matters the most.



shore@mathprojects.com  
@MathProjects

