DISTANCE DATING

WRITING EQUATIONS OF CIRCLES

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THE STORY

Fred is on a fixed budget so he has estimated the cost of dating and has calculated that he should only date girls that are a fixed distance away from his house. Fred figures that any further distance from his home will cost him too much in gasoline and any closer to his home may create a problem if he were to run into any ex-girlfriends which he dated before he was able to drive. Fred dates Ethyl who lives exactly three miles east and four miles north of Fred's home. Assume Fred's house is on the origin (0,0). At what other locations, exactly the same number of miles away from his house, can Fred potentially find a new girlfriend?

Concepts
Equation of a Circle, Pythagorean Theorem and Distance Formula

Time: 1 Hour

Materials
Straightedge, Graph paper, Student Handout

Preparation
Students need to be able to work with square roots, know the Distance Formula and are familiar with the Pythagorean Theorem.

FINDING DATES (Equation of a Circle Centered Around the Origin)

1-2. Have the students read the story. Then the students are to find six other locations (coordinate points) that are exactly the same distance as Ethyl is from Fred's home (5 miles as the "crow flies", not 7 miles. Draw segment FE directly from Fred to Ethyl.) Most students will choose points that are a combination of the original coordinates 3 and 4, like (4, 3), (-3, 4), (-4, -3), etc. Plot all these points on a graph on the board. Make sure the students label their points on their own graph paper. A few students will also have the points (0, 5), (0, -5), (5, 0) and (-5, 0). Discover how students found these other points. They more than likely used the Distance Formula or Pythagorean Theorem. This idea, $3^2 + 4^2 = 5^2$, shows that the distance from Fred's to Ethyl's house is 5 miles.

3. Unfortunately, our graph only shows a dozen possible girls that Fred can date. Fred actually wants to know ALL the locations of potential dates. Lead the students to see that if we were to plot ALL these locations, the graph would form the shape of a perfect circle. The key concept here is that the radius of the circle is the 5 mile distance to the home of any girl that Fred can date.

4. Using the discovery that the distance between the two homes is 5 miles, revisit the Pythagorean Theorem. If we let (x, y) represent the home of any girl that Fred can date, and let r represent the radius of the circle, then $a^2 + b^2 = c^2$ can be rewritten as $x^2 + y^2 = r^2$ and we have our equation of a circle!

5. This question is a contextual way of saying: for the given equation, find y when x = 2.

$$2^2 + y^2 = 5^2$$
$$y = \sqrt{21} \approx \pm 4.6$$

The equation yields two solutions for this value of x, because the circle passes through x = 2 twice! In other words, how many girls on this street can Fred date? The students can graphically see TWO.

6. The students are now to practice solving for the equation of the circle again, but this time, they substitute for y and solve for x.

$$x^2 + 1^2 = 5^2$$
$$x = \sqrt{24} \approx \pm 4.9$$

DISTANCE DATING (CONTINUED)

7. a) The streets on which Fred will be able to date only one girl are the those that are tangent to the circle ($x = -5$, $x = 5$, $y = -5$ or $y = 5$). Have the students substitute any of these linear equations into the equation of the circle so the students may see that the equation now only yields ONE solution.

   b) Fred will not be able to date a girl who lives on any street that is more than 5 miles from Fred's home. In other words, the equation will yield NO solutions for the values $x < -5$, $x > 5$, $y < -5$, or $y > 5$. Have the students substitute and see for themselves that these values produce the square root of a negative number.

FINDING NEW DATES (Equation of a Circle Centered Around a Point)

8. $(x + 4)^2 + (y - 3)^2 = 5^2$

   Have the students try this one on their own first. The new center will likely throw them for a loop. This is good, because it offers the opportunity to discuss finding the distance between two points. Let (-4, 3) represent the location of Lucy's home and (x, y) represent the location of the home of any boy that Lucy can date. If we revisit the Pythagorean theorem again, we see that for $a^2 + b^2 = c^2$, "a" is found by subtracting -4 from x: $x - (-4)$. We find "b" in a similar fashion using y: $y - 3$. By substituting 5 for the radius, we get the equation above.

FINDING MORE NEW DATES (Equation of a Circle Centered Around Any Point)

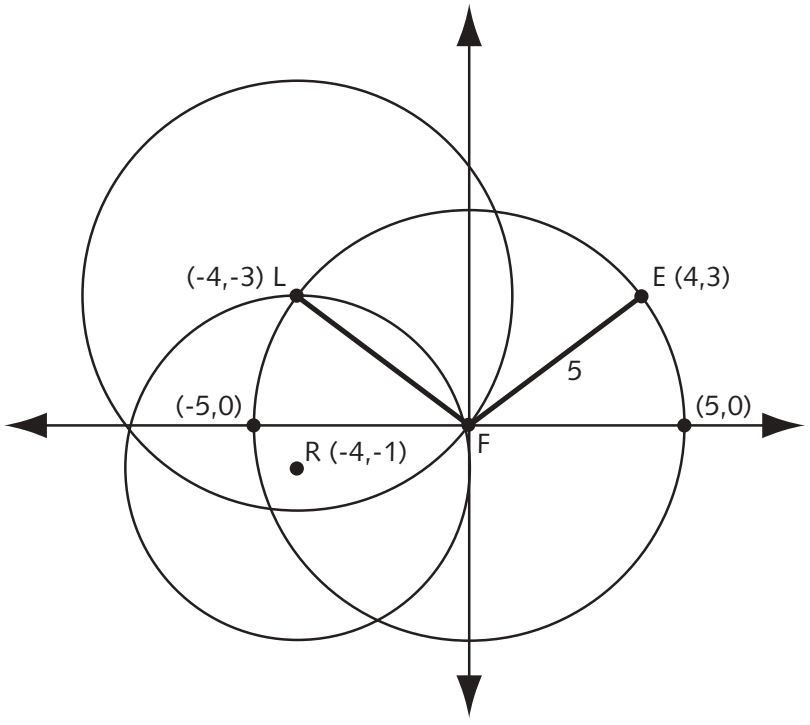
The questions in this section are used to determine if the students have the basic skills needed to write and solve equation for circles, and then to move them towards the general formula for the equation of any circle in the next section.

9. $(x + 4)^2 + (y + 1)^2 = 4^2$

10. a. $(-6 + 4)^2 + (y + 1)^2 = 4^2$
    $(y + 1)^2 = 12$
    $y = -1 \pm \sqrt{12}$
    $y \approx -1 \pm 3.5$

    b. $(x + 4)^2 + (-2 + 1)^2 = 4^2$
    $(x + 4)^2 = 15$
    $y = -4 \pm \sqrt{15}$
    $y \approx -4 \pm 3.9$

11. No, Ethyl is not a potential date for Ricky, because her coordinates do not satisfy his equation.
    $(3 + 4)^2 + (4 + 1)^2 = 7^2 + 5^2 = 74 \neq 4^2$



FINDING ANY DATE FOR ANYBODY

12. $(x - h)^2 + (y - k)^2 = r^2$

DISTANCE DATING
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FIND DATES

1. Find at least six other locations (ordered pairs) that are exactly the same distance as Ethyl is from Fred's home. Plot the points on your graph paper. How did you find these other points?

2. How far away is Ethyl from Fred? _____ miles

3. If the locations of all of Fred's possible dates were plotted, what shape would be formed?

4. Find the equation of this shape formed by all the possible locations: _____

5. Use this equation to find the locations of girlfriends that live on a north-south street that is two miles east of Fred's home. Why are there two answers?

6. Use this equation to find two unique locations of friends that live on a east-west street that is 1 mile south of Fred's home. Again, why are there two answers?

7. a) For which streets (equations) will Fred be able to date only one girl? Why?

   b) For which streets (equations) will Fred not be able to date any girls? Why?

DISTANCE DATING (CONTINUED)

FINDING NEW DATES

Lucy is a friend of Fred. She lives at the location (-4, 3). Lucy has the same budget restrictions as Fred, so she wants to date only guys 5 miles from her home.

8. Find an equation that will give the location of all of the people exactly that distance away from Lucy?

FINDING MORE NEW DATES

Ricky lives 4 miles south of Lucy. Ricky wants to date only people located 4 miles from his home.

9. What equation would you use to find all of the friends located 4 miles from Ricky?

10. a) Use this equation to find the locations of girlfriends that live on a north-south street that is two miles west of Ricky's home.

    b) Use this equation to find the locations of girlfriends that live on a east-west street that is three miles south of Ricky's home.

11. Is Ethyl a potential date for Ricky? Support your answer algebraically.

FINDING ANY DATE FOR ANYBODY

12. Write the equation of the circle that represents all the locations (x, y) that are r miles away from someone living at (h, k).