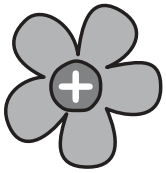
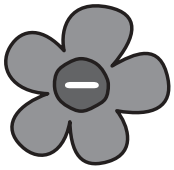


Wallflowers

adding & multiplying integers



THE NEUTRAL FIELD

As the story on the student handout shares, a girl is represented by a "positive" sign and a boy by a "negative" sign. A dancing couple is then represented by a positive paired with a negative, which equates to a "zero." Numerous couples on the dance floor are several of these zeros, which we will call "the neutral field." All the examples in this lesson start with a neutral field of five zeros, but more may be added by the students.

Concepts
Adding, subtracting, multiplying and dividing integers

Time: 2 hours. First hour, Adding/Subtracting; second hour Multiplying/Dividing.

Materials
Tile Separators, Student Handout

Preparation
You can effectively conduct this lesson by drawing positive and negative signs on the board, but tile separators make for great manipulative displays on an overhead. Purchase a small bag of tile separators at a hardware store. They look like small rubber "plus" signs. Cut the crossbars off half of them to make "minus" signs.

ADDING & SUBTRACTING INTEGERS

1. After colorfully describing the wallflower scenario (see top of first student handout), show the students the first representation: "6 - 8." In the context of the scenario: 6 girls are waiting for dance partners, then 8 girls leave. Model the diagram on the board or overhead. The initial 6 positives are the wallflowers. To represent eight girls leaving, cross-out 8 positive signs (or remove eight positive tile separators from the overhead). To do this, you remove the six female wallflowers, then two more girls from the dance floor, leaving two boys without a dance partner. The students can see the two remaining male wallflowers (negative two). Have them record their answer (-2) in arithmetic form on the handout.

2. For the second scenario, have students publicly offer possible wallflower scenarios. The correct scenario for -3 + 4 is: There are three boys waiting for partners, then four girls enter the dance. How many wallflowers are left? To draw an accurate diagram, the students must show the initial -3 as three negative signs. The four girls are drawn as four positive signs, three of which correspond to (dance with) the three negatives. This leaves one girl (positive one) remaining. Circle this remaining one (+1), then have students record their result arithmetically. (i.e. -3 + 4 = 1).

3. For #3-6, have the students make their attempts at the diagrams and written scenarios. Then have volunteers offer these on the board for discussion and correction. As usual, prod students to look for the underlying patterns and rules that will allow us to do these problems without the diagrams and/or manipulatives. Problem #7 addresses the same topic in a different context, thus it assists in checking for understanding.

SOLUTIONS

In order to protect the integrity of these lessons in the classrooms, the solutions have been removed from this version of the project. For a copy of the entire project, including all of the solutions, order ***MPJ's Ultimate Math Lessons*** *at http://www.mathprojects.com or call 1-800-247-6553 to order over the phone.*

Wallflowers

MULTIPLYING & DIVIDING INTEGERS

Multiplication is based on grouping; therefore, grouping is emphasized in this phase of the wallflower lesson. The multiplier (first value in the expression) designates the number of groups, and the multiplicand (second value) designates the number of people in each group. The sign of the multiplier designates whether the groups are arriving at (+) or leaving from (-) the dance. The sign of the multiplicand designates the gender of the group, girls (+) or boys (-). For example, 4(-2) is translated into a wallflower scenario as "4 groups of two boys enter the dance." In contrast, -2(4) is translated as "2 groups of 4 girls leave the dance."

4. In #8, use the examples above to explain the idea of grouping and the correlations between the expressions and the wallflower scenarios. Challenge the students to offer the scenario for 3(-5). In the wallflower scenario, this translates to "3 groups of 5 boys arrive at the dance." Then have the students draw the three groups of five negative signs. It helps to have the students circle each of these groups.

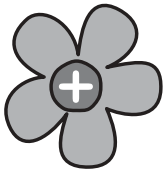
5. In #9, again challenge the students to generate the scenario for -2(3): "2 groups of 3 girls leave the dance." This is the critical point in the lesson where the meaning of the leading negative sign comes forth for the students. Arithmetically speaking, the negative two implies that we are subtracting two groups of something (in this, case subtracting 2 groups of 3 from zero). In the context of the wallflowers, emphasize that the "-2" implies that two groups are leaving the dance. Have the students draw an extra pair of dancers (+ -) on the dance floor, circle two groups of three positives, then cross out these groups. Six negatives remain.

6. In #10, the students then play with the idea of "a negative times a negative," which means to subtract a certain number of groups of a negative value. In the wallflower context, it means that a certain number of groups of boys are leaving the dance. In this particular case, it is "3 groups of 2 boys leaving." Again, have the students draw an extra pair of dancers (+ -) on the dance floor, circle three groups of two negatives, then cross out these groups. Six positives remain.

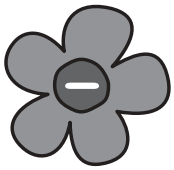
7. In #11-13 have the students make their attempts at the diagrams and written scenarios. Then have volunteers offer these on the board for discussion and correction. As with all concrete contexts, we want the students to make some abstract generalizations. After all, trying to represent "16.2 - 3.2(-4.5)" with tile separators would be a cumbersome task, if not an impossible one. Therefore, prod the students to see the general rules for multiplying integers ("two negatives make a positive" etc.), and record these in #14. Then have the students investigate the two scenarios offered in #15 a & b: $^{-12}/_{3} = -4$ and $^{-15}/_{-5} = 3$, respectively. The students should see that the rules for multiplication, also hold true for division.

SOLUTIONS

In order to protect the integrity of these lessons in the classrooms, the solutions have been removed from this version of the project. For a copy of the entire project, including all of the solutions, order ***MPJ's Ultimate Math Lessons*** *at http://www.mathprojects.com or call 1-800-247-6553 to order over the phone.*



Wallflowers
adding & subtracting integers



Algebra High School is holding its annual Integer Dance in the gymnasium. As with all dances, there is not an equal number of girls and boys, so there are always wallflowers sitting at the dance. Wallflowers are people on the perimeter of the dance floor waiting for a dance partner. The number and gender of the wallflowers can be represented by a mathematical model. (After all, this is Algebra High School.) Girls are represented by positive signs (+), and boys by negative signs (-). A dancing couple, one positive matched with one negative (+ -), equates to zero. Several dance partners make up the dance floor, mathematically termed a "neutral field." Therefore, the given diagram in #1 represents five couples dancing and six girls waiting to dance (female wallflowers).

For #1-6, a) write a scenario that may be represented by the expression, b) represent it with a "wallflower" diagram, c) evaluate the expression.

1. 6 - 8 = _____

   Scenario:_____

   _____

   _____

   + -
   + -
   + -
   + -
   + -

   + +
   + +
   + +

2. -3 + 4 = _____

   Scenario:_____

   _____

   _____

   + -
   + -
   + -
   + -
   + -

3. -3 - 4 = _____

   Scenario:_____

   _____

   _____

   + -
   + -
   + -
   + -
   + -

4. 5 - (-1) = _____

   Scenario:_____

   _____

   _____

5. -2 - (-5) = _____

   Scenario:_____

   _____

   _____

6. -8 + -4 = _____

   Scenario:_____

   _____

   _____

7. You start with $10 in your wallet, lose a bet for $16, then another bet for $4. How much money do you now have/owe?

Wallflowers
multiplying & dividing integers

As the night progresses at the Annual Integer Dance at Algebra High School, students begin to greet friends and hang out in groups. For example, the expression in #8 represents 3 groups of 5 boys coming into the dance, while #9 represents 2 groups of 3 girls leaving the dance.

For #8-13, a) write a scenario that may be represented by the expression, b) represent it with a "wallflower" diagram, c) evaluate the expression.

8. 3(-5) = _____

Scenario:_____

_____

_____

+ -
+ -
+ -
+ -
+ -

9. -2(3) = _____

Scenario:_____

_____

_____

+ -
+ -
+ -
+ -
+ -

10. -3(-2) = _____

Scenario:_____

_____

_____

+ -
+ -
+ -
+ -
+ -

11. 4 - 2(-3) = _____

Scenario:_____

_____

_____

12. 3 + 4(-2) = _____

Scenario:_____

_____

_____

13. -3(-2) - 1 = _____

Scenario:_____

_____

_____

14. Are there any rules or patterns for multiplying integers?

15. Do these rules for multiplying integers apply to division as well?
    a) You spend $12 dollars over the course of three days. You spend the same amount each day. How much do you spend each day? (Write an expression to represent this scenario)

    b) You spend $15 dollars total by spending $5 a day. How many days does it take you to spend the entire $15?