PROJECT

RISING WATER

Submitted by teachers of Sri Atmananda Memorial School, KPM Approach to Children, Atma Vidya Educational Foundation, India

OBJECTIVE

The aim of this activity is to let the students experience that equations and lines (graphs) are just different ways of expressing mathematical relationships.

Concepts
Linear relationships, dependent and independent variables, linear equations and graphs, slope, x- and y-intercepts, slope-intercept form of a linear equation

Time: 2 - 3 hours

Materials
Student handout, graph paper

Preparation
Graph of the pool example.

LESSON PLAN

Part One

Discuss a situation in which the quantities have a linear relationship. For example: think about a nearly empty swimming pool that is being filled with water. The pool begins with a water depth of 10 cm; the water rises at the rate of 3 cm per minute until it is full.

Lead the discussion in such a way that the students discover which quantities are related and how they are related — which quantity depends on the other one (in order to bring up dependent and independent variables). Often it is helpful to draw figures on the board to help the students visualize the situation. Once they have expressed the relationship in words, help them to use mathematical symbols to express the relationship. For example:

height = 3 • number of minutes or $H = 3M$ or $y = 3x$

Then let the students take several values for M (or x) and find corresponding values for H (or y). Taking these values together as ordered pairs (number of minutes, height) we get points. Have each student record these points in the table. Then plot the points on graph paper and let them find out what figure they get when all the points are connected. So we started with a relationship, turned it into an equation, then into a table of values and finally into a line! This is the crux of the lesson; students will be repeating this progression numerous times on the student handout.

At this point have the students think of several other situations in which two quantities are related. Again, have them express each relationship in words, symbols, a table of values, and a graph. Then the students should analyze the graph to see what shape is created. It may be useful if the class is divided into smaller groups to think of situations and their corresponding relationships. This gives more students a chance to present their ideas, and the teacher ends up with more situations in a shorter time. Bringing the class back together, let the students share all their ideas and split the figures they generated into two groups — those that are lines and those that are not. Then have a discussion in which the students describe what is common about all the equations whose figures turned out to be lines.

Now they know what a linear relationship is — a physical situation, a mathematical equation, a table of values, and a visual graph all at the same time!

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LESSON PLAN (continued)

Part Two
Having found the relationship, equation, table and line for each situation, it is time to compare all the linear graphs. For this part of the lesson, it is very helpful to have all the graphs drawn on large chart papers and hung in front of the class. The aim of this discussion is to use the graphs to bring out the meanings of terms like slope (gradient), x-intercept, y-intercept and the point-slope form of the equation of a line. The discussion can be precipitated with questions such as these.

In order to bring out the meaning of the term intercept:

1. What was the original height of the water in the given example? Or, at what point did the line cross the y-axis? What is the significance of that point in the given situation?

2. Did any of the lines cross through, or intercept, the x-axis? If so, what did this point represent in that particular situation? The answers to these questions can lead to a discussion on the meaning of the term 'x-intercept'.

In order to lead to a discussion on the meaning of slope:

3. In the given example, what was the height of the water at the first minute, the second minute, the third minute, and so forth? What was the difference in heights between each pair of minutes? So, what was the change in height of the water level per minute?

For the topic of slope, make sure to focus your discussion around two central issues:

1. For a given change in x, how much does y change? In other words, what is the change in y for a unit change in x?

2. Does the value of y increase or decrease for a given change in x? In other words, is the slope positive or negative?

After filling in the tables on the student handouts, the students can compare the constants in the equations to the information from the graph and form generalizations about the meaning of each constant.

NOTE: There are two reasons for having the students come up with their own situations. One reason is that the students have to make decisions, be creative and think about the concepts more openly. The other is that when the situation is their own, they identify with it; they possess it. This makes them more likely to be interested in solving it. They aren't trying to find out the solution to the teacher's problem; it is their own. They are solving something that belongs to them. So the knowledge they gain belongs to them. It isn't filtered through the teacher.

RISING WATER

Part One

1. Using the example of the pool, describe the relationship of the rising water to time.

Relationship (in words): _____

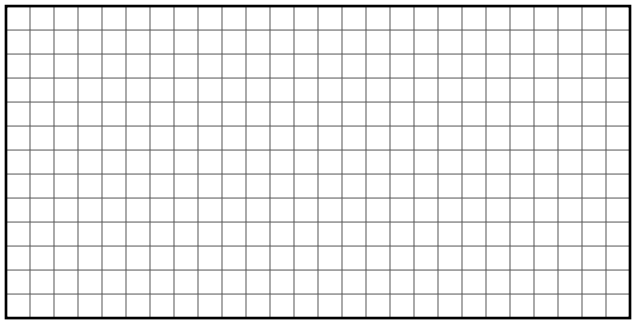
_____

_____

Relationship (using symbols): _____

Relationship (table of values):

Relationship (by graphing):



2. What were the situations and relationships that you thought of?

<u>Situation 1</u>

Relationship (in words): _____

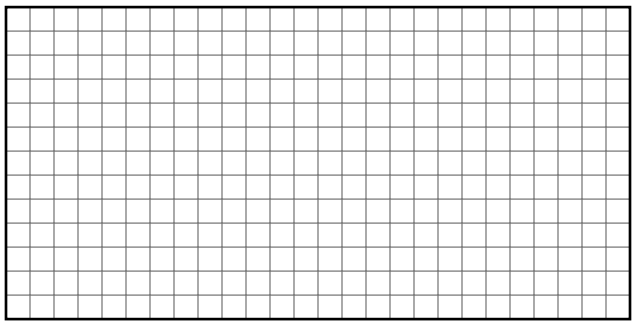
_____

_____

Relationship (using symbols): _____

Relationship (table of values):

Relationship (by graphing):



<u>Situation 2</u>

Relationship (in words): _____

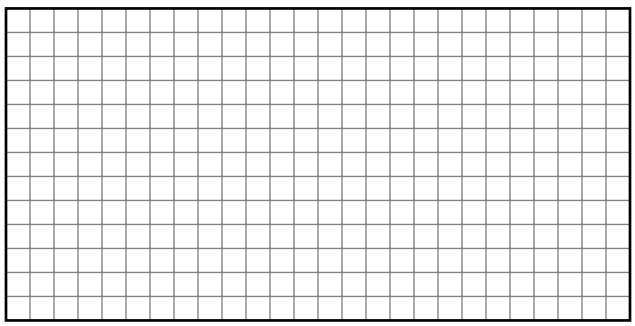
_____

_____

Relationship (using symbols): _____

Relationship (table of values):

Relationship (by graphing):



3. Fill in the following table using all the equations that came up during the discussion:

Linear Equations			
Non-Linear Equations			

4. What is a linear equation according to you? _____

_____

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PART TWO

1. Which of the situations/relationships that you thought of formed linear equations? _____

2. What is the significance of the y-intercept (the point at which the line crosses the y-axis) in each situation?

3. What is the significance of the x-intercept (the point at which the line crosses the x-axis) in each situation?

4. What is the slope in each situation? (How much does the dependent variable change for each unit change in the independent variable?) _____

5. Fill in the following table with the linear equations offered by you and the class :

Equation				
Slope				
y-intercept				

6. Explain how you could find the y-intercept by:
   a) looking at a graph _____
   b) looking at an equation _____

7. Explain how you could find the slope by:
   a) looking at a graph _____
   b) looking at an equation _____

8. On a separate sheet of paper, create two examples of linear relationships: one in which the slope is positive and another in which it is negative. Draw rough figures of those situations.

EXTRA CHALLENGE

Have the classmate to your right give you a number, m, and the classmate to your left give you another number, b. Using m as the slope and b as the y-intercept, sketch a graph and think of a situation which it could represent.